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# THE CHANGES IN THE FORM OF THE CRYSTALLINE LENS IN ACCOMMODATION

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*ABSTRACT.* The two most important existing theories of the mechanism of the accommodation of the eye are directly opposed in that they assign differently the effect of the ciliary muscle in the production of the change in the form of the crystalline lens. The theory of Helmholtz, which is the more universally accepted, states that the lens swells and increases in convexity during accommodation because the tension upon it is relaxed when the ciliary muscle contracts.

In order to explain the change of the anterior surface of the lens to a hyperbolic form in accommodation, Tscherning has propounded a theory according to which the tension of the lens is maintained when the muscle contracts, and the forms of the surfaces are altered by pressure exerted by the vitreous humour upon the periphery of the posterior lens surface.

This paper describes the measurement of the radii of curvature of both anterior and posterior surfaces of the lens, and also the movement which the apices of these surfaces make in accommodation. In two selected cases of men of the same age and having the same refractive error, considerable differences in the behaviour of the lens in accommodation were found. The results show that for a given amount of accommodation, whereas the lens surfaces in one case are more increased in curvature than in the other, their apices suffer less movement. It was also found that the surface which was most altered in curvature showed the most pronounced hyperbolic form in both relaxed and accommodated conditions. The elastic property of the lens capsule and its variations in thickness are described. The changes in the form of the lens surfaces and the differences which occur in the lenses of different individuals in accommodation are explained upon the assumption of the Helmholtz theory, by taking into account the properties of the lens capsule.

## Introduction

Few subjects in the physiology of vision have attracted more attention, or given rise to more controversy in past years, than the study of the mechanism of the accommodation of the eye. It is not my intention in this paper to enter into this controversy or to put forward a new theory, but rather to describe a number of observations and to quote some measurements made upon living eyes, in the hope that they may form a contribution, which, though modest in amount, may help towards the complete solution of this difficult problem.

The experimental work which forms the basis of this paper has extended over the past eighteen months and has necessitated a considerable amount of extraneous investigation which is not included in this account.

Before describing this work it is perhaps desirable to give a brief review of the more important existing theories of the mechanism of accommodation propounded by earlier writers and of the observations which have led up to their formulation.

In his *Traité de l'Homme* (1677) Descartes shows the diagram which is reproduced here in Fig. 1 and gives the following description of the means by which the eye is enabled to see objects at different distances: "In order to represent the point *X* distinctly it is necessary that the whole shape of this humour *NL* be changed and that it become a little flatter, as that which is marked *I*. And to represent the point *T* it is necessary it become a little more convex like that which is marked *F*." He also states that the change is brought about by the agency of the fibres by which the lens is suspended, which he describes as tendons—although he makes no mention of a muscle which operates these tendons. It is remarkable that with no experimental evidence Descartes was correct in his hypothesis.

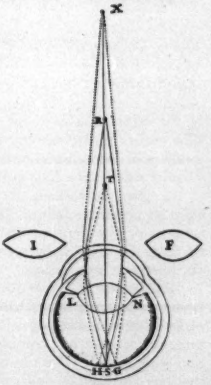


Fig. 1

In 1801 Thomas Young described a series of beautiful and conclusive experiments by which he proved that the change in power of the eye was produced by an alteration in the crystalline lens. Although many years earlier it had been suggested by Porterfield that the ciliary body contained muscle tissue, this fact had never been demonstrated; and without definite knowledge upon this point Young was unable to account for the mechanism which produced the lens changes.

It was not until 1848 that the existence of the ciliary muscle was established by the independent work of Bowman and Brücke. Max Langenbeck in 1849 and Cramer in 1851 observed and described the changes which take place, during accommodation, in the images produced by reflection at the surfaces of the lens, and thus provided a definite means of studying the behaviour of these surfaces.

Of the many theories of the mechanism of accommodation which followed upon these discoveries, that propounded by Helmholtz has survived and is at present almost universally accepted. The principle of this theory may be outlined as follows.

The anterior edge of the ciliary muscle, being inserted into the sclerotic at its junction with the cornea, is fixed, and from the physiological aspect it may be considered as the origin of the muscle. Posteriorly the muscle fibres are continuous with the choroid, which is comparatively loose in structure and contains some elastic fibres. The crystalline lens is a soft and elastic body enclosed in its capsule and suspended from the ciliary body by means of the fibres of the zonule which are attached radially to the periphery of both the anterior and posterior capsule surfaces.

Helmholtz stated that a contraction of the ciliary muscle produces a tension upon the choroid which is drawn forward, causing the inner surface of the ciliary body to make a forward and inward movement. This movement relaxes the tension which is normally exerted upon the lens and allows its surfaces to become more convex by reason of its natural elasticity.

Tscherning observed that the anterior surface of the lens assumed a hyperbolic form in accommodation, and in order to explain this fact he propounded a new theory which was in direct opposition to that of Helmholtz. He supposed that the contracting ciliary muscle exerts a *tension* upon the lens capsule through the zonule fibres, the soft outer layers of the lens being thus compressed upon the harder and more steeply curved nucleus. Tscherning claimed that such a compression would cause the centre of the lens surfaces to become more convex while their peripheral parts would be flattened. This change in figure of the surfaces had been suspected by Young and was actually demonstrated by Tscherning.

In a later revised theory which Tscherning published in 1909* he assigned an important function to the vitreous humour, which, he points out, is a tissue having a definite form and not merely a fluid. The theory may be stated very briefly as follows.

Upon contraction of the ciliary muscle a tension is exerted upon the choroid which compresses the vitreous body against the lens. Owing to the peculiar form of the vitreous and also to the fact that the anterior part of the choroid, when under tension from the ciliary muscle, will exert upon it a centripetal compression, the vitreous is pressed against the periphery of the posterior surface of the lens and not upon the centre. The anterior surface of the lens is held at its periphery by the zonule fibres. Under this compression the lens is caused to bulge in the centre; the periphery of the anterior surface becomes flattened, whilst an actual concavity is produced in a zone near the edge of the posterior surface.

In neither of these theories does Tscherning recognize a relaxation of the tension upon the zonule fibres, under the contraction of the ciliary muscle.

* M. Tscherning, *Herman v. Helmholtz et la Théorie de l'Accommodation*, Paris (1909).

Whichever of these theories be accepted, it is apparent that two factors are necessary to the production of accommodation; these are:

(1) The active contraction of the ciliary muscle.

(2) The passive properties of elasticity and flexibility of the lens which cause it to change its form under the influence of the muscle.

It is considered by most ophthalmologists of the present time that the power of the ciliary muscle is always in excess of the ability of the lens to change its form. The reduction of the amplitude of accommodation with increasing age is said to be due to the hardening of the crystalline lens; the muscle is not diminished in power. Fuchs has named the near-point of accommodation, which is measured by the usual clinical methods, the "physical near-point," i.e. the extent of the accommodation as limited by the physical properties of the crystalline lens. The measure of the maximal power of the ciliary muscle to produce accommodation in a perfectly suitable lens he has termed the "physiological near-point."

The original object of the investigation which is described herein was to determine the form of the lens in various states of accommodation. It was hoped that from measurements of a number of eyes a set of curves could be plotted which would show the thickness of the lens and the radii of its surfaces for every state of accommodation in the average eye.

Now according to the Helmholtz theory the lens removed from the dead eye is in a state of maximum accommodation, which is only limited by its own elasticity. Fuchs has stated that this elasticity is reduced with increasing age in accordance with the reduction in amplitude of accommodation. If, therefore, we measure the thickness and radii of a dead lens of known age we ought to obtain data which are in agreement with those which were found for the lens in the average living eye when accommodated by an amount equal to the maximum at the age of the dead eye.

A collection of these statistics would be useful in testing the veracity of the Helmholtz theory and also in determining the relative responsibility of lens and ciliary muscle in the reduction of the amplitude with increasing age.

A number of preliminary measurements were made upon the radius of the anterior surface of the lens in different eyes, but it was found that this value varied over a wide range for the same condition of accommodation. Two cases were found, both males of the same age and having an equal error of refraction; the anterior surfaces of the lenses in the two cases were of nearly equal radius when at rest but behaved quite differently during accommodation.

In order to determine what other differences existed, the accommodation changes were carefully studied in the two cases.

The present paper is based upon the results obtained in this investigation. It is thought that the differences which are found to exist in the behaviour of normal eyes during accommodation may in some degree account for the differences of opinion regarding the mechanism by which accommodation is produced. An acceptable theory of the mechanism must take into account these differences in behaviour of the lens and must be sufficiently comprehensive to admit of their existence.

## AUTHOR'S OBSERVATIONS

### Measurement of the Radius of Curvature of the Anterior Surface of the Crystalline Lens

#### (*a*) *Observation of the Surface and its Alteration during Accommodation*

The convex form of the anterior surface of the lens can readily be seen when it is illuminated with the light from the slit lamp. It was found that, by observing the lens obliquely with a microscope while the beam of the slit lamp was directed along the optic axis of the eye, its anterior surface could distinctly be seen to increase in curvature when the subject made a large effort of accommodation.

The direct measurement of this surface is not possible owing to the fact that observation must be made through the corneal system, and both the power of the cornea and its distance from the lens must be taken into account in estimating the true value of all observations made upon the lens.

As there appeared to be considerable difficulties in determining the effect of the cornea at the point through which the observation was made, e.g. about 1 mm. from the margin, owing to the presence of a high amount of astigmatism and the obliquity of the surface to the direction of observation, it was decided to abandon the attempt to make measurements of the lens surface by observing it in this way, although it is an interesting and, I think, new method of demonstrating the change which takes place during accommodation.

The method which was finally adopted was that used by Helmholtz and Tscherning, of making ophthalmometric measurements by means of the images produced by reflection at the lens surface.

#### (*b*) *Nature of the Images reflected from the Anterior Surface of the Lens*

Owing to the laminated structure of the lens, the succeeding layers being of increasing refractive index, the reflected image from the front surface appears badly defined by reason of the overlap of the images from surfaces immediately behind it. However, if the angle of observation be carefully selected, the central and brightest part of the image is seen to assume a granular appearance, showing that it is produced by the epithelium which covers the central parts of the lens immediately beneath the anterior capsule. As the capsule here has a thickness of only 10 to 20$\mu$, the reflections from the lens epithelium may be taken to represent the anterior surface for purposes of the measurements to be made. I have found that the quality of the images formed by reflection at the anterior lens surface varies greatly in different eyes, although the age and refractive conditions may be equal, and health of the eyes normal. While in the average case the image, even of a small bright source, is faint and nebulous and spreads over a considerable area, in a few it is almost impossible to locate owing to its faintness.

Karl Grossmann*, in a paper entitled "The Mechanism of Accommodation in

* Karl Grossmann, *Brit. Med. Journ.* (26th Sept. 1903).

Man," stated that in the particular case which he investigated it was not possible to study the form of the anterior lens surface as the reflections were not sufficiently good; and in a later work* he claims that the images are produced not by the anterior surface at all, but by reflection at the face of an irregular structure within the lens (and therefore not suitable for ophthalmometric work).

Wishing to demonstrate these images, I examined recently the eyes of a number of young men in a darkened room, by illuminating the eye with a candle flame, and was pleased to find one whose lens surface gave such well-defined reflections that the flickering form of the candle flame could distinctly be seen. So bright were the images that it was found possible, by arranging a suitable apparatus, to obtain the photographs which are reproduced in Fig. 2 (see Plate). The illumination was obtained in this case by reflecting on to the eye the light of a naked arc by means of concave mirrors, giving the effect of three sources of illumination. In order to secure the pupil large enough to exhibit all the images, a shutter was placed in front of the arc lamp, and was released simultaneously with the shutter of the camera. As the length of the exposure was only 1/5 second, it was completed before the light had caused the pupil to contract.

The bright images on the left of the pupil are reflected from the corneal surface, the central set is from the anterior surface of the lens, and the small inverted set on the right is from the posterior surface of the lens, which, acting as a concave reflecting surface, forms a real image. The photograph of the accommodated condition was made while the subject was viewing a near object placed upon the original line of sight, so that the direction of the eye was the same in both cases. It will be seen how the reflections from the anterior surface of the lens have approached one another in the accommodated state, owing to the increased convexity of the surface. The images from the posterior surface are also slightly reduced in size.

That the images from the anterior portion of the lens are produced by the actual surface, or rather the epithelium, is, I think, without doubt, as when seen with the microscope the reflection assumes a tessellated appearance which is characteristic of the single layer of epithelium. I have, however, been quite unable to find an explanation of the marked individual differences in the quality of this surface as a reflector.

### (*c*) *Measurement of the Radius*

As was pointed out by Tscherning†, the surfaces of the cornea and the front of the lens lie sufficiently near to the same plane to permit, when viewed with a telescope from a little distance, of direct comparison of their radii by comparing the relative sizes of the virtual images which they form by reflection.

In these experiments the comparison between the lens and corneal surfaces was made by using for each surface different objects which could independently

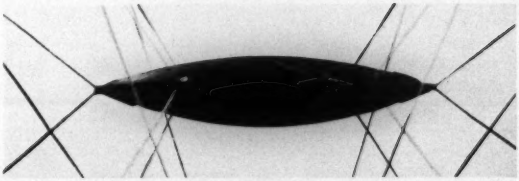
* Karl Grossmann, "Further Investigations on Accommodation," *Brit. Med. Journ.* (24th Sept. 1904).

† M. Tscherning, *Physiologic Optics*, p. 70.

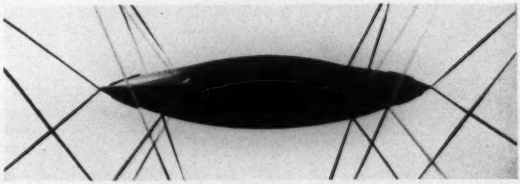
Accommodation relaxed. Accommodation 8 D.

Fig. 2

Fig. 6 (*a*) Fig. 7 (*b*)



(*a*)



(*b*)

Fig. 11

be altered in size until the image from the cornea was equal to that from the lens. Under these conditions the ratio of the apparent radius of the lens surface to the true radius of the cornea is inversely proportional to the ratio of the objects for the two respective surfaces, that is,

$$\frac{\text{Apparent radius of lens}}{\text{Radius of cornea}} = \frac{\text{Size of object for cornea}}{\text{Size of object for lens}}.$$

In order to obtain the brightest possible images from the lens surface, the object used consisted of two intensely bright motor lamps, each having a condenser placed before it to focus the image of the lamp filament on to the eye. These lanterns were clamped upon a vertical brass bar and the effective size of the object was altered by varying the distance between the lanterns. As the images from the front surface of the cornea are always bright, the object for this surface was formed by two ordinary 16 C.P. frosted lamps, each enclosed in a metal box which had a horizontal slit cut in it so that the extremities of the object for the cornea were shown as narrow horizontal lines. This pair of lamps was mounted in a similar manner to that used as the object for the lens. As these lamps were not bright, they did not give rise to apparent reflections from the lens surfaces.

The procedure adopted in order to make a comparison of the radii of the surfaces was to place the two stands carrying the lamps at a distance of one metre from the subject, whose head was placed in a suitable rest. The pairs of lamps were then adjusted in horizontal and vertical directions until their images appeared approximately central in the pupil of the observed eye. The bright lamps which were used for the lens naturally produced bright images from the cornea, in addition to those from the two lens surfaces, but by careful adjustment of the direction of the eye it was easily possible to arrange that these did not interfere with the images which were required for the measurements. It was found convenient, in order to get coincidence between the extremities of the images, to keep the object lamps for the lens fixed—usually at a separation of 130 mm.—and to adjust each of the other two lamps until their images coincided with the corresponding images from the lens surfaces. The eye was observed with a telescope of low magnification. Fig. 3 shows the disposition of the images when the adjustment was complete.

As has already been said, in order to determine the true radius of the lens surface, it is necessary to consider the optical effect of the cornea through which the reflected images are viewed. For this purpose the radius of curvature of the anterior surface of the cornea was measured with an ophthalmometer of the Javal type. The effect of the posterior surface of the cornea was neglected, a schematic value of 1·336 being taken as the refractive index of the system. It is also necessary to take into consideration the distance between the corneal surface and the lens surface—measured along the optic axis of the eye. The method of determining this value, which I have called for convenience, although not anatomically true, the "anterior chamber depth," will be described in a later part of this paper.

With this information it was possible to deduce the real radius of curvature from the apparent value, in the following manner:

Apparent distance of the centre of curvature of the lens surface from the surface of the cornea = $\bar{v}$ = apparent radius of lens surface + apparent depth of anterior chamber.

With a knowledge of the power $F$ of the corneal system, the true distance $u$ of this centre from the surface of the cornea may be calculated.

$$\bar{V}^{\bullet} = \frac{1000}{\bar{v}}, \quad \bar{U} = \bar{V} - F, \quad \bar{u} = \frac{1000}{\bar{U}}, \quad u = \bar{u} \times 1{\cdot}336.$$

The true radius of the lens surface = $u$ − the true depth of the anterior chamber.

The two cases selected for study were of the same age, 19 years. The error of refraction was equal in each case, 1 dioptre manifest hypermetropia, and both were free from astigmatism. There was found to be, however, a difference in the amplitude of accommodation. In the case H. both monocular and binocular amplitudes were 11 dioptres, while in case M. the monocular amplitude was 11·5 dioptres and the binocular amplitude 13·5 dioptres. It is usual to find at this age an excess of the binocular amplitude over the monocular of about 0·5 D.

Appearance of catoptric images in the eye:

Images of the bright lamps

Corneal surface

Anterior lens surface

Posterior lens surface

Corneal images of comparison lamps

Fig. 3

Fixation was secured by directing the subject to look steadily at the centre of a cross scratched in fine lines upon the back of a small mirror. This was placed at the correct distance from the eye for the particular test and was adjusted so that it reflected to the subject's eye light from a window which was behind his head. The lines of the cross thus appeared black against a white background. As the subjects employed were trained men, the recognition of slight irregularities in the scratched line ensured that the eye was accommodated for the fixation object.

• Letters bearing a bar superscription, thus $\bar{v}$, $\bar{V}$, refer to reduced distances in mm. and reduced vergences in dioptres.

The small mirror with the cross scratched upon the back was used as the fixation object throughout all the tests. When it was desired to observe the changes in the lens in the act of accommodation, the mirror was directed so that the image of a distant lamp behind the subject's head was seen by him in the mirror, in line with the centre of the cross. The eye which was not under observation was covered and the subject was able to accommodate either for the distant lamp or for the surface of the mirror without moving the axis of the eye.

The results of a number of observations made while the subject was exercising a definite amount of accommodation are shown in Table I. Owing to the eye in each case being hypermetropic 1 dioptre, the subjects were employing this amount of accommodation when viewing a distant object, so that observations upon the lens in the totally relaxed condition were not obtained.

Table I. *Radius of the Anterior Surface of the Lens*

CASE H., male, aged 19, R.E. hypermetropic 1 D.

Radius of the anterior surface of the cornea = 7·74 mm.
Apparent depth of anterior chamber = 3·15 mm.
True depth of anterior chamber = 3·68 mm.

| Accommodation in dioptres | Radius of lens surface in mm. |
|---|---|
| 1 (relaxed) | 11·62 |
| 2 | 10·58 |
| 2·5 | 9·90 |
| 3·5 | 9·19 |
| 5·0 | 8·50 |
| 6·0 | 7·80 |
| 7·0 | 7·55 |
| 9·0 | 6·90 |

CASE M., male, aged 19, R.E. hypermetropic 1 D.

Radius of the anterior surface of the cornea = 7·6 mm.
Apparent depth of anterior chamber = 2·8 mm.
True depth of anterior chamber = 3·3 mm.

| Accommodation in dioptres | Radius of lens surface in mm. |
|---|---|
| 1 (relaxed) | 12·0 |
| 3 | 10·06 |
| 4 | 9·2 |
| 5 | 8·5 |
| 6 | 7·6 |
| 8 | 6·2 |
| 9 | 5·5 |
| 10 | 5·0 |

The curves (Fig. 4), obtained by plotting the measured radii of curvature against the amount of accommodation exercised, show a considerable difference in the two cases. Beyond about 4 dioptres the case H. alters the curvature of the anterior surface of his lens much less than the case M. for an equal amount of accommodation.

## The Depth of the Anterior Chamber

In the study of accommodation, changes in the position of the anterior lens surface relatively to the cornea are of interest from two points of view. In the first place a change in the position of this surface will modify the dioptric power of the eye quite apart from alterations in its curvature, and so must be considered as one of the possible factors in the production of accommodation. Secondly, in considering the very interesting problem of the mechanism by which these changes in the lens are produced, the more nearly we can determine the movement of each part of the surface, the more exactly shall we be able to estimate the disposition of the forces which are responsible for the change. Thus, when Helmholtz formulated his theory of the mechanism of accommodation, it was with the knowledge that the apex of the anterior surface of the lens moved forward by about

0·4 mm. when a maximum effort was made, whereas Tscherning*, in the statement of his first theory, mentioned that he had not been able to observe a forward movement at all in this part of the surface, but had observed a backward movement of the periphery. Again in his later work Tscherning† recognizes the forward movement of the front of the lens and has modified his theory accordingly.

Helmholtz‡ considered the pupillary plane to represent the position of the anterior surface of the lens, as it undoubtedly does if the pupil be not greatly dilated. The position of this plane relatively to the summit of the cornea he determined by means of the ophthalmometer. By placing a lamp before the eye

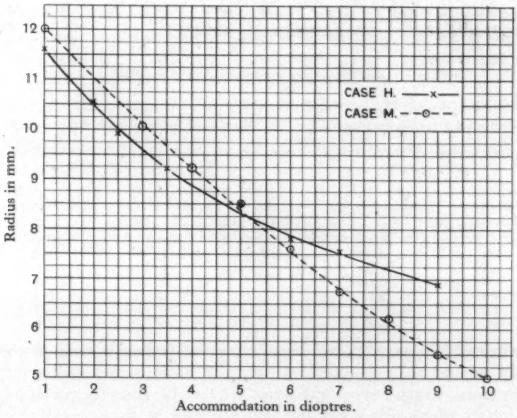



Fig. 4. Change in Radius of the Anterior Surface of the Crystalline Lens during Accommodation

and the observing telescope to one side of it, he was able to adjust the telescope and the visual axis so that the corneal reflex image of the lamp appeared to be in coincidence with the edge of the pupil. The telescope was then placed on the opposite side of the visual axis and was adjusted to bring about coincidence of the reflex with the opposite side of the pupil, the angle made by the telescope axis and the lamp at the eye being measured in each case. Knowing the distance of the reflex image behind the summit of the cornea, Helmholtz was able to calculate the apparent position of the pupillary plane, and with a knowledge of the power of the cornea could determine its true position.

* M. Tscherning, *Physiologic Optics*, p. 182.
† M. Tscherning, *Herman v. Helmholtz et la Théorie de l'Accommodation*, Paris (1909).
‡ H. v. Helmholtz, *Treatise on Physiologic Optics* (English translation), vol. 1, p. 20.

Tscherning* determined the position of the lens surface by trigonometrical methods. Using a lamp which was bright enough to produce a visible image at the surface of the lens, he placed it so that its image fell in line with the image of another and fainter lamp reflected in the cornea. Knowing the position of the centre of curvature of the cornea he was able to determine the position of the lens surface by comparing the angles made by the lamps and observing telescope with the normals to the two surfaces.

Blix† employed a novel form of ophthalmometer, the chief use of which was the accurate determination of the intra-ocular distances. His instrument consisted of two microscopes having objectives of equal focal length and mounted together so that their anterior focal points were coincident. The eyepiece of one of these was replaced by a brightly illuminated diaphragm whose image was produced at the point where the two axes met. When this image was focussed upon a surface, an observer looking into the other microscope would see the image in focus and placed centrally in the microscope field. The necessary movement of the apparatus to bring the image reflected from different surfaces successively into coincidence with a cross-wire in the field of the microscope was equal to the apparent distances between the surfaces.

In the present case the measurements were made by means of the combined corneal microscope and slit lamp‡. This instrument produces an intensely bright image of an illuminated slit which is focussed on to the tissues of the eye. Observation is made with a microscope, which is so mounted that it is constantly directed toward the focus of the illuminating system. Thus the plane upon which the slit image is focussed will appear centrally in the field of the microscope.

The principle employed here is similar to that used by Blix except that, whereas that author observed the reflected images from the surfaces when the light was focussed upon them, the slit lamp renders the transparent tissues visible by the light which they scatter and so permits the position of their surfaces to be measured.

It was not until after I had published the description of the combined corneal microscope and slit lamp that the work of Blix on the measurement of intra-ocular distances was kindly brought to my notice by Mr Priestley Smith.

As it was desired to make this measurement as nearly as possible along the optic axis of the eye, it was necessary to determine first the angle between the optic axis and the visual axis (angle $\alpha$), in order that the subject's visual axis might be directed so as to bring his optic axis into line with the instrument. This angle was measured by means of the apparatus which was used to study the reflected images from the lens and corneal surfaces. The lamps in this case were placed vertically, one above and one below the observing telescope. The subject's vision was then adjusted in the horizontal direction until the images from the various surfaces appeared to be all placed upon a vertical line which passed through the

* M. Tscherning, *Physiologic Optics*, p. 67.

† H. v. Helmholtz, *Treatise on Physiological Optics* (English translation), vol. 1, p. 326.

‡ E. F. Fincham, "A New Form of Corneal Microscope with Combined Slit Lamp Illuminating Device," *Trans. Opt. Soc.* 25 (1923–24) 113.

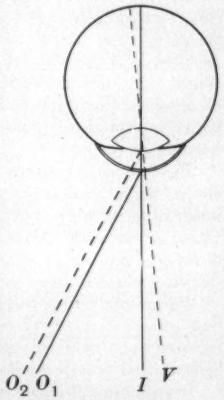
centre of the pupil. Under these conditions the optic axis of the eye was in alignment with the axis of the telescope, and the angle between this axis and the visual axis was measured.

In making the measurement of the anterior chamber it was found to be necessary to hold the head of the subject in a rest which was strutted in such a way that it permitted of no movement whatever. The head-rests usually supplied by the makers of ophthalmic apparatus have been found to allow a quite considerable forward and backward movement of the head, which renders them useless for this purpose.

Having fixed the head, the scale upon the focussing slide of the slit lamp was set to zero and the instrument moved up towards the eye until the image of the anterior surface of the cornea appeared coincident with the cross-wire in the microscope eyepiece. During this manipulation the subject was directed to view an object placed at the right distance to his nasal side of the slit lamp to bring his optic axis into line with the latter.

As soon as this adjustment was completed the instrument was racked forward from this first position of coincidence with the corneal surface until the image of the front surface of the lens was coincident with the microscope cross-wire. The scale and vernier upon the focussing slide showed the distance which the instrument had travelled forward. This distance is equal to the apparent depth of the anterior chamber.

In order to obtain the true depth from the measured value, it is sufficient to know the refractive index of the medium and the power of the cornea. The directions of the illuminating beam and of the lines of observation during this experiment are shown in Fig. 5.




$I$, Line of illumination
$O_1$, First line of observation
$O_2$, Second line of observation
$V$, Visual axis

Fig. 5

By placing the corneal microscope at right angles to the slit lamp and adjusting the latter so that the light entered the eye along the optic axis and focussed upon the anterior surface of the lens, a quite appreciable forward movement of the pole of the lens could be seen when the subject made an effort of accommodation. A micrometer scale was then placed in the eyepiece of the microscope with a view to making direct measurements of this movement of the lens surface. Although the readings obtained by this method appeared to be quite consistent, the difficulty in estimating the effect of the extremely peripheral part of the cornea, through which the observations were made, prevented the determination of the true movement.

Having measured the depth of the anterior chamber in the two cases under observation, by the method already described, a series of measurements was made

upon these cases while they were exercising definite amounts of accommodation. The results are shown in Table II.

Table II.

| Case H. | | | Case M. | | |
|---|---|---|---|---|---|
| Accommodation in dioptres | Depth of anterior chamber in mm. | Movement of lens surface in mm. | Accommodation in dioptres | Depth of anterior chamber in mm. | Movement of lens surface in mm. |
| 1 (relaxed) | 3·68 | — | 1 (relaxed) | 3·33 | — |
| 4 | 3·55 | 0·13 | 4 | 3·22 | 0·11 |
| 6 | 3·50 | 0·18 | 6 | 3·17 | 0·16 |
| 9 | 3·34 | 0·34 | 9 | 3·06 | 0·27 |

Since studying these two cases I have examined the eye of another man of the same age. Here the depth of the chamber with accommodation relaxed was found to be 3·45 mm., and with 8 dioptres of accommodation in use was 2·79 mm., showing that the lens surface moved forward by 0·66 mm.

Tscherning* has published the measurements of the movement of the anterior pole of the lens made by a number of workers. Some of these have measured many eyes and it may be of interest here to mention the extreme figures and the average of each observer's results (see Table III). Scholer and Mandelstamm, Adamuck and Woinow, Reich and Knapp worked with the method of Helmholtz, while Besio and Maklakoff employed Tscherning's method. The measurements by Saunte were obtained with an ophthalmometer of special construction designed by him in collaboration with Tscherning.

Table III.

| Observer | Forward movement of anterior pole of lens in mm. | | |
|---|---|---|---|
| | Greatest | Least | Average |
| Helmholtz | 0·4 | 0·3 | 0·35 |
| Scholer and Mandelstamm | 0·5 | 0·2 | 0·35 |
| Adamuck and Woinow | 0·7 | 0·2 | 0·45 |
| Reich | 0·4 | 0·3 | 0·366 |
| Knapp | 0·7 | 0·6 | 0·65 |
| Tscherning | 0 | — | — |
| Besio | 0·4 | 0·4 | 0·4 |
| Saunte | 0·4 | 0·2 | 0·275 |
| Maklakoff | 2·1 | 0·3 | 1·15 |

Except where the observer has not mentioned the amount of accommodation, the figures quoted above were obtained when the subjects accommodated by an amount between 7 and 10 dioptres.

In the examination of 40 eyes Maklakoff found a movement of over 1 mm. in no less than 18 cases, and in six of these the movement amounted to 2 mm.

Although the fact of the forward movement of the anterior pole of the lens appears to be firmly established, there is some difficulty in reconciling the results obtained by Maklakoff with those of other workers. Especially is this so when

* M. Tscherning, *Herman v. Helmholtz et la Théorie de l'Accommodation*, Paris (1909).

we realize that he used the same method as Tscherning, who found no movement, and Besio, who recorded a movement of 0·4 mm.

### The Form of the Anterior Surface of the Lens

The observations and measurements recorded in the preceding paragraphs refer only to that area of the lens surface which is exposed by the pupil and in the accommodated eye under normal illumination does not exceed 3 mm. in diameter. Although only this exposed area is effective optically, a knowledge of the form and behaviour of the more peripheral zones is of great importance in considering the changes which take place during accommodation, and the mechanism which brings about these changes.

#### (*a*) *Historical*

A number of the early anatomists describe the crystalline lens as having hyperbolic surfaces. Descartes in his *Traité de l'Homme* (1677) describes the form of the "crystalline humour" as "similar to that of those lenses which I have described in the *Traité de la Dioptrique*, by means of which all the rays which come from a certain point reassemble at another certain point." There seems to be little doubt that this reference is to what we now know as the Cartesian lens which Descartes first described in his *Traité de la Dioptrique* (1637). Kepler also in his *Ad Vitellionem Paralipomena* (1604) depicts the crystalline lens with hyperbolic surfaces. Cherubin d'Orleans, however, in *La Dioptrique Oculaire* (1671) argues that the hyperbolic form of the crystalline can serve no useful purpose, and states his belief that the surfaces are spherical.

As Thomas Young* first proved experimentally that the crystalline lens was instrumental in producing accommodation, so he showed by means of his optometer that the amplitude of accommodation diminishes towards the periphery of the pupil, from which he concluded that the lens surfaces assumed the form of a hyperboloid during accommodation.

Stadfelt†, using Foucault's method, measured the spherical aberration of the crystalline lens removed from a dead eye and showed that the central area of about 4 mm. in diameter was approximately aplanatic. This he found to be surrounded by a zone extending to about 3·5 mm. from the axis, in which the aberration was over-corrected by about 2 dioptres. He explained this over-correction as due to the diminution of the refractive index towards the periphery and not to a distortion of the surfaces.

Tscherning repeated the experiments of Young showing that the amplitude of accommodation was greatest for the central part of the pupil, and demonstrated that the normal spherical aberration of the eye became over-corrected during accommodation. This he succeeded in proving to be due to a change in the figure of the lens, particularly of its anterior surface. In order to show this, he arranged three bright lamps upon a horizontal arc so that their images, reflected from the

* T. Young, "On the Mechanism of the Eye," *Phil. Trans. Roy. Soc.* (1801).
† M. Tscherning, *Physiologic Optics*, p. 107.

anterior surface of the lens, fell close to the upper border of the dilated pupil. When the accommodation was relaxed, these three images were seen to lie upon a straight line or upon a curve slightly concave towards the centre. During accommodation the middle image was moved more than the others so that they then made a line which was convex towards the centre. From this experiment Tscherning concluded that under accommodation the centre of the surface became more steeply curved while the periphery suffered flattening, thus producing a hyperbolic surface.

It was chiefly upon this fact that Tscherning based his original theory of the mechanism, supposing that the periphery of the surface was flattened by the traction which the zonule exerted upon it, thus pulling the central part of the cortex back upon the harder and more steeply curved nucleus.

### (*b*) *Author's Observations*

For the purpose of observing these phenomena in the eyes of the two cases that were the subjects of the present experiments, the pupils were dilated with homatropine and cocaine, and the observations were made before any considerable paralysis of the accommodation had become manifest. The illumination was supplied by focussing upon the eye the images of a "Pointolite" lamp produced by five small concave mirrors, which were mounted upon a metal arc held vertically before the eye. Observation was made with a low-power telescope as was the case when measurements were made of the radius of the lens surface. The apparent diameter of the pupil in the unaccommodated state was 8 mm. and rather less during accommodation.

Figs. 6 and 7 (see Plate) are reproductions of careful records made at the time of the observation; the row of large faint images in each case is produced by reflection at the front surface of the lens. In case H., when the accommodation was relaxed, the row of images from the anterior surface of the lens formed a straight line close to the edge of the pupil; a figure of this is not given here. When the eye accommodated 6 D, the row of images became smaller and moved toward the centre of the pupil, still, however, remaining straight. By rotating the eye slightly, the images could be brought back to the edge of the pupil, where they were larger and were seen to form a line which was slightly convex towards the centre of the pupil, as is shown in Fig. 6. It appears from this observation that in this particular eye the anterior surface of the lens approaches closely to the spherical form when at rest and is not greatly distorted under the influence of accommodation.

The eye of the case M. showed a remarkable difference in this respect. While the subject was viewing a distant object in such a position that the images from the front surface of the lens fell near to the margin of the pupil, the line of images assumed a marked curve, those at the top and bottom extremities being splayed out toward the periphery as is shown in Fig. 7 *a*. When the subject accommodated 6 D without altering the direction of the visual axis, the lens images became smaller and moved toward the centre of the pupil, arranging themselves in the order shown in Fig. 7 *b*. It will be noticed here that the three most central images fall upon a

straight line, showing that the portion of the surface producing them, which is about 3 mm. in diameter, is approximately spherical. The images at the two extremes show a remarkable splaying, characteristic of reflection from a conical surface.

From these observations it appears that the anterior surface of the lens in this eye in the unaccommodated state varies from the spherical form by a flattening, commencing probably beyond about 2 mm. from the axis. It would be difficult to say to what extent this surface approaches the true hyperboloid. When this eye accommodates, the lens surface takes on the form of a shallow cone having its apex rounded into a spherical curve extending over an area about 3 mm. in diameter. Normally it is only this spherical portion which is exposed by the pupil.

## The Peripheral Depth of the Anterior Chamber

In an endeavour to verify the results of the observations upon the form of the lens surface and its behaviour during accommodation, the next piece of work which was undertaken was the determination of the depth of the anterior chamber at a point near the periphery of the lens. A knowledge of this distance, considered in conjunction with the previously measured position of the anterior pole of the lens and the curvature of the central portion of the surface, will enable us to make an approximate graphical representation of the form of the surface.

As the observations upon the periphery of the lens must be made through a peripheral part of the cornea, and as also the position of the lens surface must be measured from the corneal surface, it is first necessary to measure the curvature of the cornea at this point and to determine its position relatively to the vertex of the surface.

### (*a*) *Determination of the Form of the Corneal Surface*

As is well known, the cornea varies considerably from the spherical curve and, although in this case it was required to know only its horizontal contour, it was necessary to explore it piecemeal from the centre to the periphery.

The usual forms of ophthalmometers were not suitable for this purpose as it is not possible with these instruments to confine the measurements to a sufficiently small area of surface. The method which was used was that suggested by Dr Drysdale* for measuring the radius of curvature of small surfaces. For this purpose the corneal microscope was employed because its mounting allowed it to be suitably manipulated before the observed eye. A vertical illuminator was attached to the microscope and to this was fitted vertically a tube which carried a small electric lamp at its open end. The length of the tube was such that the lamp was held at a distance from the objective equal to the image distance of the objective.

In making the measurement of the radius, the microscope was brought up to the eye and its direction and position so adjusted that a sharply focussed image of

* C. V. Drysdale, "On a Simple Direct Method of Measuring the Curvatures of Small Lenses," *Trans. Opt. Soc.* 2 (1900–01) 1.

the filament of the lamp was seen upon the surface of the cornea by an observer looking into the microscope. The instrument was then racked forward until another image of the filament was brought into focus. This second position is found when the light from the objective is striking the corneal surface normally; consequently the distance by which the microscope travels forward between the two observations is a measurement of the radius of curvature of the surface. In order to confine the measurement to a small area of the surface, a diaphragm was placed over the objective of the microscope, thus limiting the illuminated area of the cornea, when the microscope was in its second position, to less than 2 mm. in diameter.

A series of measurements was made in this way, extending from the centre of the cornea to the limbus on the temporal side. This, of course, necessitated the rotation of the eye in order that the microscope should be directed normally to each successive section of the cornea. The angle which the axis of the microscope made with the visual axis of the eye in each position was recorded, and from these results, together with the radii of the various parts of the curve, a graphical construction of the horizontal contour of the surface was made.

### (*b*) *Measurement of the Peripheral Depth of the Anterior Chamber*

It was found, when the pupil was dilated to a diameter of 8 mm., that a beam of light which passed normally through the cornea at a point 1·5 mm. from the limbus would strike the lens surface near the margin of the pupil. This position was therefore selected for the purpose of the measurement. The radius of the cornea for the horizontal meridian at this point was 13 mm. in the case H. and 12·3 mm. in the case M.

The depth of the chamber was measured at this point in the two cases—the method used being the same as that described for the measurement of the central depth. To secure that the light was incident normally to the corneal surface, the instrument was adjusted so that the beam of light reflected back from the cornea fell upon the centre of the projection lens of the illuminating tube. The microscope was placed close to the illuminating tube so that observation and illumination were made through as nearly as possible the same part of the cornea.

### (*c*) *Construction of the Form of the Lens Surface from the Measurements obtained*

From the experiments which have been described we have obtained the following data regarding the anterior surfaces of the cornea and lens in the eye at rest:

(1) Form of the corneal surface.
(2) Distances of the anterior pole of the lens from the apex of the cornea. (Depth of anterior chamber.)
(3) Distance of a point on the lens surface from a point on the corneal surface at 1·5 mm. from the limbus, measured along the normal to the corneal surface. (Peripheral depth of anterior chamber.)
(4) Radius of curvature of the central part of the lens surface.

In Fig. 8 the approximate form and disposition of the anterior surfaces of the cornea and lens have been constructed from these data for both the cases which were studied. As the radius of curvature of the periphery of the lens surface was unknown, this part is represented by a straight line, the direction of which is governed by the measured peripheral depth of the anterior chamber. The broken line in each case shows the position which would be occupied by a spherical surface having a curvature equal to that which was found at the central part of the lens.

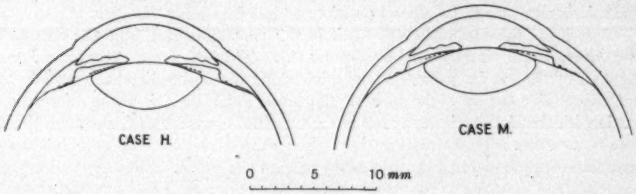



Fig. 8

An anterior zonule fibre passing tangentially from the periphery of the lens surface to the inner surface of the ciliary body is represented by the straight line on either side. This and the remaining parts of the diagrams were copied from a microscopic section of the anterior portion of the human eye.

### Increase of Thickness of the Lens in Accommodation

The question of the change in thickness of the lens is important in the consideration of the mechanism of accommodation, as upon our knowledge of this must depend very largely our judgment of the function of the ciliary muscle.

It was upon his observation of an increase in the lens thickness that Helmholtz based his theory of a relaxation of the tension upon the lens during the contraction of the muscle. He recorded a maximum forward movement of the anterior surface of 0·4 mm. while the posterior surface remained stationary.

In Tscherning's statement of his original theory he records that the lens is not increased in thickness.

In measuring the apparent thickness of the lens in the two cases which were the subject of the present investigation, the corneal microscope was used in the same manner as has been described for the measurement of the depth of the anterior chamber. In the calculation of the true thickness it was necessary to consider the cornea and the anterior lens surface as one lens system through which the observations were made. The change in the depth of the chamber and the radius of the anterior surface of the lens were taken into account in order to ascertain the power of the combined system in the accommodated state. The lens substance was considered as a homogeneous medium having an effective mean index of refraction of 1·413. The results of the measurements are given in Table IV.

Table IV. *Author's Measurements*

| | Accommodation in dioptres | Central thickness of lens in mm. | Increase in thickness of lens in mm. |
|---|---|---|---|
| Case H. | 1 | 3·66 | 0·58 |
| | 9 | 4·24 | |
| Case M. | 1 | 3·84 | 0·36 |
| | 9 | 4·20 | |

### Movement of Posterior Pole of Lens

By comparison of the above figures with the record of the movement of the anterior pole of the lens (change in depth of anterior chamber) we see that the backward movement of the posterior surface is partly responsible for the increase in thickness. An increase in accommodation of 8 D is accompanied by a backward movement of the posterior pole of the lens of 0·24 mm. in case H., and 0·09 mm. in case M.

Although the main object of the present paper is to make a comparison of the differences which occur in these two selected cases, it is important to notice that Maklakoff recorded a very wide range of readings for the movement of the posterior surface of the lens. His results show that, whereas in a few cases the surface remained stationary, in twenty cases the movement was backward and in eight cases forward. The greatest backward movement recorded was 0·8 mm. and the greatest forward movement 1·3 mm.

The results of the majority of other workers who have found a movement of this surface show it to be very small and in the backward direction.

### The Radius of Curvature of the Posterior Surface of the Lens (Central Area)

The exact determination of the radius of the posterior surface of the lens, like the measurement of the thickness of that body, is difficult owing to our deficiency of knowledge of the refractive properties of the lens substance. In this case the measurements obtained from the preceding observations were made use of in the calculation of the true radius from the apparent value which was measured. Again schematic values of 1·336 and 1·413 were taken as the indices of the aqueous and lens respectively.

The measurement of the apparent radius was made by means of the comparison ophthalmometry method which was used for the measurement of the radius of the anterior surface. Owing to marked distortion effects it was necessary to keep the object lamps close together so that the measurements were made upon a small central portion of the surface. The results are given in Table V.

Table V.

| | Accommodation in dioptres | Radius of posterior surface of lens in mm. | Change in radius in mm. |
|---|---|---|---|
| Case H. | 1 | 5·18 | 0·13 |
| | 9 | 5·05 | |
| Case M. | 1 | 5·74 | 0·87 |
| | 9 | 4·87 | |

From these results it will be seen that, although the posterior surface alters very slightly, its behaviour is similar to that of the anterior surface; that is to say that in the two cases which were studied the surfaces which altered most in radius during accommodation suffered the least movement of their apices. Case H. shows the greater movement of both front and back surfaces, whereas the surfaces in case M. show the greater increase in curvature.

### Change in the Equatorial Diameter of the Lens

The observation of the change of the diameter of the lens during accommodation is probably the best criterion of the nature of the effect which the ciliary muscle exerts upon it. It is readily apparent that an increase in diameter would signify the application of tension upon the lens capsule by the zonule fibres, while a reduction in diameter would be consistent with a relaxation of tension, allowing the lens to assume a more convex form by reason of its elasticity.

An opportunity of making this observation upon the living eye very rarely presents itself owing to the margin of the lens being masked by the iris. Grossmann* was fortunate in finding a case of complete aniridia in which a considerable amplitude of accommodation existed. This case was hypermetropic and so, in order to obtain complete relaxation of the ciliary muscle, homatropine was instilled. The equatorial diameter of the lens was then found to be 12·25 mm. Under the influence of eserine, which produced a maximum contraction of the muscle, the lens measured only 10·25 mm. in diameter.

The difference in these figures appears to be unduly large, but is probably accounted for by the fact that unfortunately Grossmann does not appear to have taken into account the effect of the cornea, which at this peripheral position is probably considerable.

Hess† made exact measurements of the lenses, after death, of eyes which shortly before death had been treated with atropine and eserine. In order to make the examination, the eye was divided at the equator and an enlarged tracing was made by means of the drawing camera of the plan of the lens and surrounding structures as viewed from the back. In a case aged 20 the lens of the eye which had been treated with eserine was found to be 0·5 mm. smaller in diameter than that in the other eye of the same case which had been treated with atropine. Other observers have recorded that a decrease in the diameter of the lens takes place when a fresh eye is divided at the equator and an electrical stimulus is applied to the ciliary region.

The author has studied a case in which about 4 mm. of the circumference of the lens was exposed by a large coloboma. By suitable illumination the edge of the lens, the course and insertions of a number of anterior and posterior zonule fibres, and the tips of three of the ciliary processes could be seen. Although the examina-

* Grossmann, *loc. cit.*

† C. Hess, "Beiträge sur Kenntniss accommodativer Aenderungen im Menschenauge," *Archiv für Augenheilkunde*, 65 (1910).

tion was made with a microscope employing a magnification of 20 diameters, no movement of any of these structures was apparent when the subject made an effort of accommodation of 10 dioptres.

There appears to be, however, ample evidence that the equatorial diameter of the lens is reduced during accommodation. I have been unable to find a record of an observation of a contrary change.

## The Lens Capsule

### (*a*) *Elasticity*

The lens is enclosed in a continuous capsule of a transparent, homogeneous, and very elastic material. This last property, which has an important effect upon the form of the lens, can easily be demonstrated. If a microscopic section of the lens be treated with orcein or any of the specific elastin stains, the capsule is seen to be deeply stained although the lens substance is scarcely affected. Its elasticity is also shown by the way in which a strip of it, when torn from the lens, rolls up at the edges.

Bowman*, in describing this membrane, gives the following account. "Its elasticity, which is one of its most remarkable properties, is evinced by a curious experiment which presented itself to me accidentally when I was occupied in a series of researches into the anatomy of these parts. When removed from the eye, and placed in water, the lens imbibes fluid through its capsule, which thereby becomes distended and separated from the contained lens, being raised in the form of a vesicle. If it be taken from the water and punctured with a needle, the fluid is ejected with violence by the resilience of the distended capsule, which instantly contracts to its former bulk, and grasps the lens closely."

This statement by Bowman, that when the capsule returns to its original form it grasps the lens closely, is of particular interest in regard to another observation, which, although only a small matter in itself, appears to me to be of considerable significance in the study of the mechanism of accommodation. If a small incision be made in the lens capsule of a fresh eye it is seen to gape open, the soft lens substance being protruded through the aperture to form a decided blister upon the surface. This effect is not due to gravitation of the lens substance as it takes place when the incised surface is uppermost; neither is it caused by the tension exerted upon the capsule by the zonule, because it can be seen in a lens which has been removed from the eye. It points to the fact that the elastic capsule is kept in a state of distension by the lens substance, upon which it is constantly exerting pressure. The effect produced by making an incision in the capsule is consistent with such a condition.

* William Bowman, *Lectures on the Parts Concerned in the Operations on the Eye* (1849), p. 62.

### (*b*) *Thickness*

In a further description of the capsule Bowman mentions the fact, which has been missed by many anatomists, that its thickness is not uniform. He states that the central region of the anterior part is four or five times the thickness of the posterior part.

A comprehensive table of measurements of the thickness of this membrane at various parts and in eyes of different ages is given in Salzmann's *Anatomy and Histology of the Human Eyeball*. From this table there appears to be no consistent change in the thickness of the capsule with increasing age, although great differences appear in different individuals. The measurements of a few cases selected from Salzmann's table are given in Table VI.

Table VI.

| Age | Anterior pole | Maximum of anterior surface | Equator | Posterior pole | Maximum of posterior surface |
|---|---|---|---|---|---|
| years | μ | μ | μ | μ | μ |
| 15 | 9 | 14 | 14 | 3 | 23 |
| 19 | 12 | 23 | 17 | 3 | 26 |
| 26 | 10 | 18 | 10 | 3 | 17 |
| 32 | 12 | 16 | 16 | 2·3 | 21 |
| 35 | 14 | 21 | 17 | 4 | 23 |
| 36 | 9 | 21 | 16 | 3·4 | 22 |

Salzmann states that the capsule of the anterior surface has its maximum thickness in a zone at 3 mm. from the pole, whereas the capsule of the posterior surface, which is extremely thin over the greater part, reaches its maximum near to the equator of the lens.

Table VII shows the values for the capsule thicknesses which I have obtained by the measurement of sections of three eyes of unknown age.

Table VII. *Thickness of Lens Capsule*

| | Pole μ | 2 mm. from pole μ | 3 mm. from pole μ |
|---|---|---|---|
| Anterior | 10 | 17·5 | 13·5 |
| Posterior | 2·5 | 7 | 17 |
| Anterior | 21·5 | 28 | 24 |
| Posterior | 4 | 8 | 19·5 |
| Anterior | 15 | 22 | 18 |
| Posterior | 2 | 4 | 8 |

In each of these sections the maximum thickness of the anterior capsule was found to be nearer to the pole than it is stated to be by Salzmann. In other respects the figures show a general agreement.

A diagrammatic representation of a section of an average lens capsule is given in Fig. 9. In this the relative thickness of the capsule is magnified 100 times.

## CONCLUSIONS

In the consideration of the theory of the mechanism of accommodation we must accept apparently the experimental evidence of the increase of the central thickness and decrease of the equatorial diameter of the crystalline lens. These facts appear to indicate the general truth of the theory of Helmholtz, that the contraction of the ciliary muscle causes a relaxation of the tension exerted upon the lens by the zonule, thus allowing it to swell and increase in power.

Tscherning and his followers, who constitute the chief school of opposition to this theory, appear to have based their objections chiefly upon the fact that the surface of the accommodated lens is distorted from the spherical form, a change which is not explained by a simple relaxation of the tension. The theories of Tscherning were constructed in order to explain this change in the figure of the surfaces, which Helmholtz had not taken into account in the formulation of his theory.

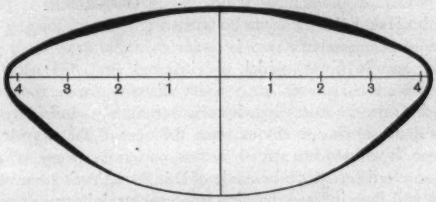



Fig. 9

We have seen, from the observations upon the two selected cases which are described in this paper, that not only is this change in the figure different in different eyes, but also that the increase in central thickness of the lens is not proportional to the increase in the curvature of its surfaces. In these particular cases it was found that, for an equal increase in accommodation, the surface which suffered the greatest change in curvature showed the least movement of its apex. It was this surface also which showed the greatest variation from the spherical curve in both the relaxed and accommodated conditions. If, then, we are to accept the Helmholtz theory, some factor must be found which will explain the change in the figure of the anterior surface and will also permit the existence of the differences of behaviour in different eyes.

I would suggest that this factor is to be found in the combination of the elastic nature and varying thickness of the lens capsule. We have seen, from the little experiment of making an incision in the capsule, that this membrane is kept in a distended condition by the lens substance, upon which it exerts pressure. If, therefore, parts of the capsule are weaker, they will be distended more readily by the pressure from within, and will consequently cause the surface to be more steeply curved in these parts. Thicker and stronger parts of the capsule will resist

distension and so cause the surface to be flattened. The lenticonus described by Tscherning may therefore be accounted for in the same way as the keratoconus, which is caused by the weakening of the centre of the cornea.

In Fig. 10 are reproduced two remarkable photographs by Pflugk showing the form of the lens in the eyes of a monkey. The photograph on the left shows the lens in its natural state, and that on the right the form after treatment with eserine. By comparison of these photographs with the diagram of the lens capsule (Fig. 9) we see that the most convex parts of the surface correspond to the thinnest parts of the capsule; the more flat or concave parts are found where the capsule is thickest. It is also noticeable that the greatest concavity occurs near the periphery of the posterior surface where the change in the thickness of the capsule is most acute.

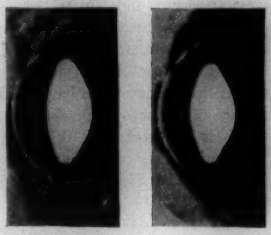


Fig. 10

Assuming the Helmholtz theory to be true, the lens in the unaccommodated state is under tension by the zonule fibres, which are inserted into the capsule upon the anterior and posterior surfaces at a short distance from the equator. This tension upon the capsule causes it, where its strength is sufficient, to flatten the surfaces of the lens. Owing to the extreme thinness of the capsule covering the posterior surface, it will tend to stretch in that part rather than compress the lens substance. If an elastic envelope be made of this form, having one surface thicker than the other, and then inflated, traction upon the periphery will cause flattening of the thicker surface; the other will remain unchanged.

It thus appears that the posterior surface of the lens capsule is distended to almost its maximum curvature when the accommodation is relaxed, i.e. when it is under tension by the zonule; it consequently does not increase much in curvature when the tension is relaxed during accommodation.

The movement of the apex of this surface may be explained by the general swelling of the lens, produced by the reduction in its equatorial diameter. As this movement is spread over a relatively large area, the change in curvature is very small.

The tension upon the anterior surface of the capsule will cause its curvature to be reduced and, if the variation in thickness be not great or the diameter of the thinner central area be small, the form of the surface will be approximately spherical. The anterior surface of the lens in case H. appeared to be spherical when unaccommodated. If, however, the centre of the capsule be thin or this area be relatively large in diameter, it will be stretched under the influence of the tension and so will cause less compression of the lens substance in this part. The resultant surface would show a greater curvature in the centre than towards the periphery. Such a form was found to exist in the anterior surface of the lens in case M. in the unaccommodated state.

When, during accommodation, the tension upon the surface is relaxed, the capsule becomes distended by the pressure of the lens substance, the distension being most apparent where the capsule is weakest. In a case in which the variation in the thickness of the anterior capsule is considerable, the distension of the central part of the surface will be relatively great and, as the bulging is limited to a small area, the curvature of this part will be greatly increased in proportion to the amount of movement of the apex of the surface.

I have endeavoured to illustrate this effect of the varying thickness of the capsule by means of a model which is illustrated in Fig. 11 (see Plate). A circular elastic bag was made by cementing two sheets of india-rubber together at their edges. One sheet was about three times the thickness of the other. The variations in thickness in the lens capsule were imitated by cementing additional rings of rubber to the internal surface of the bag, in the same positions in which the increased thicknesses occur in the human lens capsule.

Some difficulty was experienced in obtaining smooth and regular surfaces to the bag owing to the drag of these rings of additional thickness, but this was overcome by vulcanizing it in a mould of the required form; a pair of concave glass-working tools were employed as a mould. The vulcanizing process caused the surfaces to be spherical and equal in curvature. When the bag was gently inflated, the thinner surface became the more distended, the other assuming a hyperbolic curve similar to that which has been found to exist in the anterior surface of the crystalline lens. This india-rubber model lens was suspended by means of thin tapes which were attached to the peripheries of both surfaces in such a manner that tension could be applied equally and in an approximately tangential direction to the surfaces.

In the photographs the thicker surface is uppermost. It will be seen that under the influence of tension both surfaces were nearly spherical (Fig. 11 *a*), the thinner surface having the greater curvature. When the tension was relaxed the thicker surface assumed a hyperbolic form (Fig. 11 *b*) which was similar to the accommodated condition of the anterior surface of the human lens. The centre of this surface became more steeply curved, while the periphery was flattened. The thinner surface remained practically unchanged in curvature although the relaxation of tension was equal for both surfaces.

The behaviour of the anterior surface of the lens in case M. was found to be of this order, i.e. in the unaccommodated condition the curvature was greater in the centre than toward the periphery, and during accommodation, although the apex made only a relatively small forward movement, the curvature at the centre increased considerably and the periphery became flattened.

The anterior surface in case H., which in the unaccommodated state is approximately spherical, shows less increase in central curvature, but a greater movement of the apex, than does the surface in case M. for an equal increase in accommodation, and is also much less distorted from the spherical curve. This behaviour can be explained upon the assumption that the capsule covering the anterior surface of the lens in case H. is more uniform in thickness. The distension which takes

place during accommodation will then be more uniform and consequently the increase in curvature will be less in proportion to the forward movement of the apex, and the surface will not be greatly distorted from its spherical form. It would appear from this reasoning that a greater contraction of the ciliary muscle and therefore greater relaxation of tension was necessary in case H. to produce a given increase in accommodation.

The values for the amplitude of accommodation in the two subjects, obtained by means of the optometer, tend to show that this is the case. Although the ages and refractive errors were equal in the two subjects, both the monocular and binocular amplitudes in case H. were 11 D, whereas in case M. the monocular amplitude was 11·5 D and the binocular amplitude 13·5 D. The difference between the monocular and binocular amplitudes is dependent upon the reserve of ciliary muscle power.

In this connexion it is important to record that an instillation of homatropine caused an appreciable reduction of the amplitude in case H. in a few minutes, whereas in case M. its effect was not noticeable until a considerably larger time had elapsed. If the paralysing effect of the drug be equal in both cases, the difference in the time taken to produce a manifest result points to the fact that case M. has a greater excess of muscle force over what is needed to produce his maximum accommodation. Assuming that the total power of the muscle is equal in both cases, we must conclude that a greater muscular effort is necessary in case H. to produce a given increase in accommodation.

Since the completion of the observations which have been described, I have examined the lenses of freshly-killed sheep in order to compare the form of the surface and the thickness of the capsule with those in the human lens. The lenses of sheep, like those of oxen, horses, rabbits, etc., are more bulbous in form than the human lens, that is to say, the thickness is greater in comparison with the equatorial diameter, and the surfaces are apparently more spherical.

The cornea and iris were removed from three fresh eyes of sheep and the form of the anterior surface of the lens was studied in each case by holding above it a white card bearing a straight black line, the image of which was reflected in the lens surface. This image was observed through a small aperture which was in the centre of the line, this arrangement being similar to the familiar Placido Disc. By bringing the image of the line from the centre of the lens surface toward the edge, it was seen to suffer no apparent distortion until the extreme margin was reached, where the curvature is much increased owing to the rounding off of the edge of the lens. These observations indicated that in the three sheep lenses which were studied the anterior surface was approximately spherical up to within a short distance from the equator.

These three lenses were then hardened with Zenker's solution and sections were cut from them by means of the celloidin process. Measurement of the anterior capsule, under a magnification of 1000 diameters, showed the thickness in one case to be uniform, while in the others it gradually increased from the centre toward the periphery. These measurements are given in Table VIII.

Table VIII.

| Lens Number | Pole μ | 1 mm. from pole μ | 2 mm. from pole μ | 3 mm. from pole μ | 4 mm. from pole μ | 5 mm. from pole μ |
|---|---|---|---|---|---|---|
| 1 | 16 | 16 | 16 | 16 | 16 | — |
| 2 | 17 | 17 | 18 | 18·5 | 20 | 21 |
| 3 | 18 | 18·5 | 18·5 | 20·5 | 21 | 22·5 |

The absence of the ring of increased thickness surrounded by a thinner zone, which is found in the human capsule, is of particular interest when considered in regard to the fact that the anterior surface of the sheep lenses approximated to the spherical form.

Owing to the extreme thinness of the lens capsule there appears to be no method of measuring its thickness in the living eye. We are therefore without a means of corroborating the theory of its function in accommodation and must leave it at present in the nature of a hypothesis.

In concluding I wish to express my indebtedness to Mr Priestley Smith for the loan of much of the existing literature upon this subject, which would otherwise have been difficult of access; my thanks are also due to Mr C. H. Walters for his assistance in many of the observations, and to Mr H. H. Emsley for many valuable suggestions and for the facilities of experiment which he has freely granted.

## DISCUSSION

Mr Priestley Smith (communicated): It seems to me that Mr Fincham shows us a reasonable way out of a very perplexing conflict of evidence. We nearly all agree with Helmholtz in assuming that contraction of the ciliary muscle slackens the zonule and tends to lessen the diameter of the lens. Indeed, when the late C. Hess showed that eserine often makes the lens tremulous, and allows it to be displaced by gravity, and when he succeeded later in examining lenses shortly after death which were fixed in the conditions of rest and accommodation, doubt on that point seemed impossible. Yet Tscherning, following Thomas Young, had proved beyond question that the accommodated lens is *not* more globular than the unaccommodated, as the Helmholtz theory suggests, but that it takes on a hyperbolic form, and had declared that this change of form was incompatible with the alleged slackening of the zonule. His inference was unsafe, as I was able to show to the Ophthalmological Society*. A hoop of flexible steel was shaped to resemble on a large scale the contour of the lens in profile, and was stiffened in certain regions and left highly flexible in others in such a way that when stretched laterally (zonule tense) it had the form of the lens in repose, and when not so stretched (zonule slackened) it took the hyperbolic accommodation form. I give

* Priestley Smith, *Trans. Ophth. Soc.* vol. 19 (Nov. 1898); also *Ophth. Rev.* (Dec. 1898), p. 344.

the original diagram of the model here (Fig. A); the dotted line shows the form assumed under lateral traction of the zonule (accommodation relaxed), the continuous line the form when lateral traction ceased (accommodation exerted). Truly this steel hoop had little in common with a living lens, but its accommodation-form was fairly life-like, and was induced by *slackening* the lateral pull—the Helmholtz theory; it suggested that the real accommodation change of form may conceivably depend on a peculiar distribution of flexibility in the lens capsule.

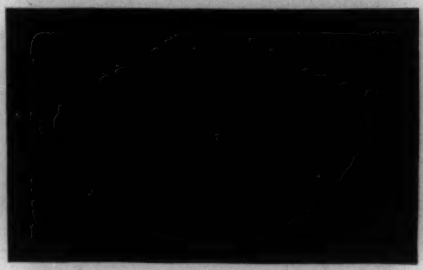


Fig. A

I see that the author's observations have confirmed the work of others on several points, but that they have not left the question where it was when he took it up. In certain eyes he has found that the capsule varies much in thickness in the several zones, and that these variations, taken together with its known elasticity and tension, appear to account well for the changes of form which actually occur, and thus to reconcile facts which have hitherto seemed at variance. I hope Mr Fincham will be able to pursue the question further.

Dr Margaret Dobson: In cases of cataract, presbyopia is often normal or it may be in excess of the age or myopia may develop, reducing the amount of presbyopia. The elastic property of the lens and its non-uniformity in thickness, as demonstrated by Mr Fincham, may account for some of the accommodative anomalies in cataract. In senile (nuclear) and deep cataracts, where the lens capsule still retains its elastic property, which it can exert upon that part of the underlying cortex which is not involved, myopia may develop owing to an actual increase in the curvature of the anterior capsule, although the increased refractive power of a denser nucleus is also a factor. Cataracts where striae are only present in the extreme periphery of the lens—that part which Mr Fincham has proved moves least, or not at all, during accommodation—have a normal presbyopia. When presbyopia is in excess of the patient's age, the cataract often involves the anterior of the capsule of the lens, thus destroying its elastic power. The thickened ring, 3 mm. from the pole, is often the seat of disease and, as demonstrated by Mr Fincham,

cannot now exert any pressure to form an increased curvature in the anterior (pupillary) portion of the lens. In this way a presbyopia in excess of the patient's age may result.

Mr C. L. Redding: I am glad that Mr Fincham has emphasized the marked variation, in different eyes, in the quality of the images formed by reflection at the anterior surfaces of the crystalline lens. These images are particularly useful in investigating the mechanism of accommodation; I have often observed the marked differences to which Mr Fincham has referred, and would be interested to know the cause of these variations.

Appreciating as I do the difficulty of locating these images in certain eyes, Mr Fincham is to be congratulated not only for having discovered an individual whose lens gave such well-defined reflections, but more especially for being, I think, the first to make use of photography to record and demonstrate the relative sizes and positions of these images with accommodation relaxed and exerted. The photographs which Mr Fincham has obtained are unique, and very considerable skill and care must have been used to obtain such remarkably good results.

With regard to the change in the equatorial diameter of the lens during accommodation, I have made observations with the slit lamp on the case of coloboma of the iris referred to in the paper, and could not detect any movement of the suspensory ligament fibres nor of the edge of the lens section. Assuming that the diameter of the lens is reduced by 0·5 mm. during accommodation, its edge, under a magnification of twenty diameters, should appear to move about 5 mm. towards its axis, and it is rather strange that no movement could be observed.

Although the paper deals principally with the changes in the form of the crystalline lens in accommodation, it would be interesting to know if Mr Fincham has, during his observations with the slit lamp, observed any change in the apparent brightness of the illuminated section of the lens during accommodation. The brightness of this area does not depend only on refractive index, but a change in the brightness might indicate that the refractive index of the lens altered during accommodation.

According to certain theories the vitreous humour is assumed to be in close contact with, and give support to, the posterior surface of the lens during accommodation and Mr Fincham might throw further light upon the mechanism of accommodation by giving the results of his observations on the post-lental space. Mr Fincham has called attention to the slit lamp method of demonstrating the changes in the form of the lens during accommodation; changes in the post-lental space can be observed by the same method, and although, strictly speaking, not within the scope of the present paper, I am sure that Mr Fincham's remarks upon the changes in the post-lental space during accommodation would be interesting.

Mr Fincham: Mr Priestley Smith has shown that many years ago he was able to prove, by means of a model, that the hyperbolic form which the lens assumes in accommodation is not inconsistent with a relaxation of the tension upon the fibres of the zonule. It is surprising that the publication of his work was not followed

by a careful study of the capsule and the lens substance, with a view to discovering the properties of these structures which are suggested by the model.

There is, I think, an important difference between the model which Mr Priestley Smith showed and my own. In his model a strip only of the capsule is shown and this appears to assume a hyperbolic form of its own accord when the traction upon it is relaxed. The steel spring is not stretched. In the suggestion which I have put forward the capsule is held in a distended condition by the lens substance, the distension being greatest where the capsule is weakest. The model which I showed was distended with air and was air-tight. I do not think that a model made from strip steel would show the absence of change in the posterior surface during tension and slackening of the lateral pull. This effect can be shown by a similar model made from strip india-rubber wrapped round a slab of soft gelatine having the form of a section of the lens. If the india-rubber on one surface be twice the thickness of the other, this surface only will be flattened when lateral tension is applied; the thin surface will remain practically unchanged.

I should like to take this opportunity of thanking Mr Priestley Smith for kindly lending for inclusion in these *Transactions* the original block illustrating his model.

The points mentioned by Dr Dobson are of great interest and contain valuable suggestions. Without some knowledge of the changes in the refractive index of the lens substance it would, I think, be difficult to determine with any degree of precision the cause of acquired myopia or the presbyopic changes in the various forms of cataract. Although a nuclear cataract may not so seriously affect the amplitude of accommodation as would a change involving the hardening of the cortex and the capsule, the softness and elasticity of the entire lens substance appears to be necessary to the retention of the theory of Helmholtz. Some useful information could, I think, be gained by studying the accommodative changes in cataract cases of various types during the progress of the disease.

In reply to Mr Redding, I would say that I have not observed a change in the brightness of the illuminated section of the lens in accommodation when under observation with the corneal microscope and slit lamp. Such a change may exist, but would, I think, necessitate special methods of observation in order to determine it. The brightness of the section of the media as illuminated with the slit lamp does not depend upon the refractive index but upon the homogeneity of the medium. A perfectly homogeneous medium would appear dark under these conditions, unless it produced fluorescence. The crystalline lens, although a fluorescent body, appears bright in the slit lamp beam chiefly by reason of its heterogeneous fibrous structure. The differences in tension which occur in accommodation may produce a difference in the closeness of packing of the lens fibres sufficient to cause a change in the light-scattering properties of that body. It would be interesting if this change could be demonstrated by means of the slit lamp as suggested by Mr Redding.

There appears to be some difference of opinion regarding the relation of the anterior surface of the vitreous humour to the posterior surface of the lens. Microscopic observations of the living eye under illumination from the slit lamp seem to show that the vitreous is attached to the lens surface only at a narrow zone

situated about 2 mm. from the equator of the lens. The space within this zone, known as the post-lental space, varies in depth. It is probably never as great as 1 mm., while in a few cases the vitreous surface appears to rest in contact with the lens capsule. In the average case a space, which does not appear luminous in the beam, is seen behind the lens; this is succeeded by the surface of the vitreous as a faint grey curtain which is fibrous in structure.

I have mentioned in the paper that the posterior surface of the lens can be seen to make a backward movement in some cases during accommodation. So far as I have been able to ascertain, no change takes place in the depth of the post-lental space, that is to say, the anterior surface of the vitreous moves backward with the lens surface.

I have succeeded, with the eyes of sheep and bullocks, in inflating this space with a coloured fluid, which showed that the connexion between lens and vitreous was water-tight. This would mean that the posterior surface of the lens is in contact with an enclosed chamber of fluid which, from the mechanical aspect, is equivalent to its being supported by the vitreous humour as it is stated to be according to some of the theories of accommodation.

# THE FUNDAMENTAL LAW OF ANNEALING

By F. W. PRESTON

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[Communicated by Mr F. Twyman, F.R.S.]

*MS. received, 6th April*, 1925. *Read and discussed, 14th May*, 1925.

*ABSTRACT.* The fundamental law of annealing that the rate of decrease of stress varies as the square of the stress present is deduced from first principles by means of dimensional analysis, assuming

(1) that the rate of decrease of stress depends only on the stress present and the viscosity;

(2) that the rate of decrease of stress depends on the stress present, the viscosity and the rigidity of the material.

It is shown that the law suggested as an empirical relation by Adams and Williamson is obtainable directly from dimensional reasoning.

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THE process of annealing glass, as ordinarily understood, is complicated by the fact that the annealing has to be effected at a high temperature, while the glass is ultimately needed at room temperature. The technological problem therefore is a dual one: the problem of annealing proper, and the problem of getting the glass down to room temperature.

As a physical process, however, annealing consists fundamentally of a process that may be carried out at constant temperature; pitch may be annealed at room temperature for example. It consists in taking a piece of glass (say) which exhibits internal stress, and allowing that stress to decrease spontaneously at constant temperature by an actual slow viscous flow of the parts of the specimen.

The rate of decrease of stress $\left(-\frac{df}{dt}\right)$, therefore, will presumably depend on the stress $f$, and on the viscosity $\eta$. It is conceivable that it may also depend on the elasticity or compressibility of the glass.

Assuming at first that it depends only on $f$ and $\eta$, then the method of dimensional analysis* gives

$$-\frac{df}{dt}=\phi\,(f,\eta)=B\frac{f^2}{\eta},$$

where $B$ is a dimensionless constant. Integration gives

$$\frac{1}{f}-\frac{1}{f_0}=\frac{B}{\eta}t \qquad \ldots(1),$$

$f_0$ being the initial stress at $t=0$.

* Edgar Buckingham, *Phil. Mag.* 42 (1921) 696; *Journ. Amer. Soc. Mech. Eng.* (June 1915).

This is the law found experimentally by Adams and Williamson* and by them regarded as purely empirical, adopted by Hampton† and criticized by Smith‡ as devoid of a physical basis.

Adams and Williamson expressed their results in the form

$$\frac{1}{\Delta_n} - \frac{1}{(\Delta_n)_0} = At,$$

where $\Delta_n$ is the birefringence, which is proportional to the stress and $A$ is a constant of the material. Their method of expressing it rather obscures the physical meaning of the equation: in particular it disguises the fact that it is in the nature of a reciprocal of the viscosity, or a measure of the "*fluidity*" of the material§. Nevertheless, the Adams and Williamson result—obtained without any theoretical guidance—and the result in equation (1)—obtained on purely theoretical principles—are identical, and each justifies the other.

Let us now see what would have been the effect of including the elastic properties of the material in our dimensional equation. We may choose to work with Young's modulus $E$, the shear modulus or rigidity $G$, or the compressibility $K$, or with all of these, but, as they are interconnected by means of Poisson's ratio, which is a dimensionless number, it is sufficient for our purpose to work with only one. As the process of viscous flow is a shearing action, we should instinctively choose the rigidity $G$.

The dimensional equation then becomes

$$\phi\left(\frac{df}{dt} . \eta . f . G\right) = 0.$$

The dimensionless equation obtained by the operation of the $\pi$ theorem of Buckingham is

$$\phi\left(\frac{\eta \dfrac{df}{dt}}{f^2} \cdot \frac{f}{G}\right) = 0$$

or

$$-\frac{df}{dt} = \frac{f^2}{\eta}\phi\left(\frac{f}{G}\right) \qquad \ldots(2).$$

The form of the function $\phi\left(\frac{f}{G}\right)$ is entirely unknown, and must be discovered experimentally. Adams and Williamson seem to have discovered that, at the elevated temperatures of the ordinary annealing range, $\phi\left(\frac{f}{G}\right)$ reduces, for all practical purposes, to a constant.

The interpretation to be placed on this fact is not perfectly certain: the most

* Adams and Williamson, *Phys. Rev.* 14 (1919) 98; *Journ. Opt. Soc. Amer.* 4 (July 1920) 219.

† Hampton, *Trans. Opt. Soc.* 26 (1924–25) 14.

‡ Smith, discussion of Mr Hampton's paper, *loc. cit.*

§ This point did not, however, escape the notice of Adams and Williamson; see *Journ. Franklin Inst.* 190 (1920) 620.

natural one is that the rigidity $G$ has nothing to do with the case; that at these temperatures the whole matter of annealing is in the hands of the viscosity, as Twyman seems to have suspected.

On the other hand it seems certain that at lower temperatures the rigidity must be considered, and this possibly is the explanation of Adams and Williamson's results* at very low temperatures, where it is shown that the relation between $\frac{1}{f}$ and $t$ is no longer linear. At any rate their results *could* be explained by the assumption that at low temperatures $\phi\left(\frac{f}{G}\right)$ reduces not to a constant, but to a function of $f$ which has to be discovered from the experimental results.

Other interpretations of a more speculative character are conceivable, and lead to interesting possibilities; but, as the experimental data to verify the conclusions do not at present exist, it would be out of place to discuss the subject further here.

The present paper is written to show that no exception can be taken to Adams and Williamson's formula on dimensional grounds. If it is to be disproved, or modified, or extended, it must be as the result of further experimental work.

It is desirable to call attention to the fact that Trouton and Andrews† so far back as 1904 found a very similar law for the rate of release of strain in *pitch* and specifically called attention to the fact that the law is *not* logarithmic (exponential) but rather resembles that of a rectangular hyperbola (i.e. is of the Adams and Williamson type). The data given in their paper are, however, somewhat meagre, though they corroborate those obtained by Adams and Williamson for glass many years later.

Twyman‡ states quite definitely that $-df/dt$ is proportional to $f$, as Clerk Maxwell§ assumed. On the other hand Trouton and Andrews‖ in the case of pitch and Adams and Williamson in the case of glass find that $-df/dt$ is proportional to $f^2$. Neither of these laws is physically impossible.

Clerk Maxwell wrote $-df/dt = f/T$, where $T$ is a certain time that he dubbed the "relaxation-time." The equation is dimensionally homogeneous and would be entirely satisfactory and useful if it had been shown that $T$ were a true constant of the material, i.e. that it were independent of $f$. But nobody knows whether there is a physical property such as the "relaxation-time" of a material that is a true constant of the material.

On the other hand there is some considerable body of evidence that for most materials there is a certain constant known as the "coefficient of viscosity" that is, to a first approximation, a true constant of the material, i.e. independent of the stress. This coefficient is the ratio of the stress to the resulting rate of shear.

* Adams and Williamson, *Journ. Franklin Inst.* 190 (1920) 622.
† Trouton and Andrews, *Phil. Mag.* 7 (1904) 347.
‡ Twyman, *Journ. Soc. Glass Techn.* 1 (1917) 61.
§ Clerk Maxwell, *Phil. Mag.* (Feb. 1868).
‖ Trouton and Andrews, *loc. cit.*

Now if we assume with Twyman or Clerk Maxwell that the rate of annealing ($-df/dt$) depends solely on the stress $f$ and viscosity $\eta$, and if we further assume that $\eta$ is a true constant, then the Adams and Williamson law, $-df/dt = Bf^2$ ($\propto f^2/\eta$), follows immediately and inevitably from dimensional considerations without any knowledge whatever of the physics of the process.

Clerk Maxwell's assumption, on the other hand, would imply that, if his $T$ were a true constant, the coefficient of viscosity is proportional to the stress, a most unlikely result, though of course not utterly impossible.

# THE DIMENSIONAL ACCURACY OF MR HAMPTON'S PAPER ON "THE ANNEALING OF GLASS"

By F. W. PRESTON

(The Standard Plate Glass Company, Butler, Pennsylvania.)

[Communicated by Mr F. Twyman, F.R.S.]

*MS. received, 16th April, 1925. Read and discussed, 15th May, 1925.*

*ABSTRACT.* No corrections are made in Mr Hampton's results, and it is shown that they are based on sound reasoning. The dimensional requirements are straightened out, however, and, together with the theoretical justification of the Adams and Williamson formula for the fundamental law of annealing (see previous paper)*, this seems to put the theory of the annealing process on a fairly sound basis, and to justify in every way the practical conclusions of Mr Hampton's paper.

THE present brief paper is provoked somewhat belatedly by the recent arrival in my hands of Mr Hampton's paper† and Mr Smith's comments thereon in the discussion.

Mr Hampton's paper is rather hard reading and confusing to a physicist by the apparent lack of dimensional homogeneity in the equations, and by occasional failures to define clearly what the various quantities are. A careful investigation of the paper shows, however, complete consistency of thought, and the paper can be remodelled on dimensional lines with very little trouble. It seems desirable that this should be done, as outlined below, since then, in combination with the dimensional accuracy of the Adams and Williamson formula established in the previous paper, we have a fairly sound theoretical basis for our treatment of annealing problems.

Two assumptions remain, viz. that the coefficient of expansion of glass is constant over the annealing range, and that Twyman's law connecting viscosity with temperature accurately represents the facts over the same range‡.

In my previous paper I have shown a theoretical basis for the law $-\frac{df}{dt} = \gamma f^2$, where $f$ is stress and $\gamma$ is fluidity§.

* Preston, "The Fundamental Law of Annealing," *Trans. Opt. Soc.* 26 (1924–25) 270.

† Hampton, *Trans. Opt. Soc.* 26 (1924–25) 14.

‡ Twyman stated this law to be correct, but gave no data. The writer has found a precisely similar law to hold with other highly viscous fluids. It is known that it does not hold over an indefinitely extended range, but it appears to be a good law for the range involved. Theoretical deduction of the complete law will probably be impossible for many years to come.

§ $\gamma$ is probably the fluidity in absolute C.G.S. units, but may be a multiple of this. For the sake of conciseness I shall treat it as if it were the fluidity in absolute units: the effects of this assumption will be obvious to the reader and do not affect the general agreement.

Hampton does not work with the stress, but with the "optical strain," expressed in wave-lengths per cm., and denoted in his paper by the symbol $s$. It is a dimensionless quantity, and is defined as

$$s = \mu_x - \mu_y = \frac{nl}{t},$$

where $n$ is the number of wave-lengths of phase difference, $l$ is the wave-length in cm., and $t$ is the length of the light path in the specimen in cm. $\mu_x - \mu_y$, or less correctly $\mu_e - \mu_0$, I shall call $\tilde{\mu}$ (= birefringence).

Let $\sigma$ be the stress optical coefficient; then $\tilde{\mu} = \sigma f$ and $\tilde{\mu}$ is Hampton's $s$. Then Hampton's equation $\frac{ds_t}{dt} = -\lambda s_t^2$ becomes

$$\frac{d\tilde{\mu}}{dt} = -\lambda \tilde{\mu}^2.$$

Hence $\lambda$ in Hampton's equation has dimensions $T^{-1}$ (as is otherwise obvious) and since $\frac{d\tilde{\mu}}{dt} = \sigma \frac{df}{dt}$, while $\tilde{\mu}^2 = \sigma^2 f^2$, we get

$$\sigma \frac{df}{dt} = -\lambda \sigma^2 f^2 \quad \text{or} \quad -\lambda = \frac{1}{\sigma} \cdot \frac{df}{dt} \cdot \frac{1}{f^2}.$$

But
$$-\frac{df}{dt} \cdot \frac{1}{f^2} = \gamma$$

Hence
$$\lambda = \frac{\gamma}{\sigma} = \frac{1}{\eta\sigma}.$$

Now in Hampton's equation (1), $\lambda_\theta = \lambda_0 2^{\frac{\theta - \theta_0}{d}}$, $\lambda_\theta$ is the "velocity constant for relief of stress" at temperature $\theta$, and $\lambda_0$ is the velocity constant at temperature $\theta_0$. This law is true whatever value we give to $\theta_0$, but we shall adopt Hampton's definition that $\theta_0$ is that temperature that makes $\lambda_0 = 1$, which, since $\lambda = \frac{1}{\eta\sigma}$, is that temperature at which the stress optical coefficient is numerically equal to the reciprocal of the viscosity, or numerically equal to the fluidity. Since $\lambda$ is not a dimensionless constant, the value of $\theta_0$ will change according as time is measured in seconds or in minutes. Hampton has adopted a somewhat extraordinary system of units of mass, length, and time, but he has used them consistently throughout: they are as follows:

$L$ = Unit of length = 1 cm.
$T$ = ,, time = 1 minute
$M$ = ,, mass = $g$ kilograms,

where $g$ is the acceleration due to gravity in cm. per min. per min.

$$= 981 \times 3600 \times 1000$$
$$= 3{\cdot}54 \times 10^9 \text{ grams.}$$

Viscosity measured in this system will be $\frac{60}{3{\cdot}54 \times 10^9} = 1{\cdot}7 \times 10^{-8}$ times as great as in the absolute C.G.S. system. Since $\sigma$ (from Hampton's Table II) is of the order ·005, $\theta_0$ is a temperature at which the viscosity is of the order $10^{10}$ absolute C.G.S. units.

Returning now to Hampton's paper, p. 16; equations (4) and (5) are correct as they stand, $s_t$ and $s_0$ being defined as "optical strain" or birefringence, and $\lambda$ having the dimensions $T^{-1}$ and being equal to $\frac{1}{\eta\sigma}$.

In equation (6), $s_0 = pc$, $p$ has the dimensions $\theta T^{-1}$ while $c$ has the dimensions $\theta^{-1} T$.

Everything is here in order till we get to the middle of p. 17, where for the sake of dimensional homogeneity $\lambda = \lambda_0 2^{\frac{n-pt}{d}}$ should be written in place of $\lambda = 2^{\frac{n-pt}{d}}$. This $\lambda_0$, numerically equal to 1 (one per minute), being retained throughout, $cd\lambda_0$ will occur in equations (10), (11), and (12) in place of $cd$ whenever the latter appears. $cd\lambda_0$ is dimensionless.

On p. 18 it is understood that Young's modulus is in kilograms per square centimetre, the same as $F$*. Equation (13),

$$F = \frac{ap}{6K(e-f)}(b^2 - 3x^2),$$

is then dimensionally homogeneous. $K$ has dimensions $L^2T^{-1}$.

In Table II, p. 19, the first column $\frac{a}{6K(e-f)} = J$ (say) has the dimensions $ML^{-3}T^{-1}\theta^{-1}$, the second column (stress optical coefficient or $\sigma$) has the dimensions $M^{-1}LT^2$, and their product $D$ in the third column has the dimensions $L^{-2}T\theta^{-1}$.

Putting $x = 0$ in equation (13), we get $F = Jpb^2$ and, since Hampton's

$$s = \tilde{\mu} = \sigma F,$$

as previously shown,

$$\tilde{\mu} = \sigma F = J\sigma pb^2 = Dpb^2,$$

or, as Hampton writes it,

$$s_0 = Dpb^2 \qquad \text{.....(15)}$$

and

$$c = Db^2 \qquad \text{.....(16)},$$

which results are of course identical with Hampton's and dimensionally satisfactory.

The equations at the bottom of p. 19 to be dimensionally satisfactory should read

$$\frac{s}{s_0} = \frac{\frac{Db^2 d\lambda_0}{\log_e 2} 2^{\frac{n}{d}}}{1 + \frac{Db^2 d\lambda_0}{\log_e 2} 2^{\frac{n}{d}}}$$

with mental reservations that $\lambda_0$ is numerically equal to unity.

* The introduction of a new $f$ in $(e-f)$ is here unfortunate, but as it cannot be helped, being a legacy from Adams and Williamson, I have changed my $f$ (meaning stress) to $F$.

It seems, therefore, that Hampton's results are entirely correct and his numerical values calculated from entirely satisfactory premises. The apparent inconsistency of his equations arises solely from the fact that the zero of temperature ($\theta_0$) is taken arbitrarily (though very conveniently) at a point where $\lambda$ is numerically equal to unity, so that $\lambda_0$ disappears from a number of equations where it is wanted for the sake of dimensional homogeneity.

In the second part of Hampton's paper (Reannealing: pp. 23–28) the same difficulty occurs, so that $c$ (a new constant on p. 24), which should be dimensionless, appears to have the dimensions of time, though its numerical value is correct when it comes to working out the tables.

While, therefore, I agree with Mr Smith that "Mr Hampton's mathematical discussion as it stands seems unconvincing," it is so only superficially. So far as I can see, although it is not perfectly expressed, it shows a clear insight into the real issues at stake.

On the other hand I agree with Mr Hampton that Mr Smith's statement that "deductions based on the Adams and Williamson formula are decisively disproved by the observations" needs explanation. Adams and Williamson evidently found that it fitted the facts, and they had nothing but the facts to guide them. Hampton states that to the best of his knowledge it is a good formula, and his experience is considerable. I have shown in the previous paper that the formula has a sound theoretical basis or rather that its basis is sound so far as our present theoretical knowledge goes.

If Mr Smith knows of observations decisively contradicting the Adams and Williamson results, it would be interesting to have them. I can only imagine that the explanation thereof must be that the compressibility or elasticity of the glass plays in some cases a more important part than Adams and Williamson experienced (see previous paper).

## DISCUSSION

Mr T. Smith: I am glad that my remarks in the discussion on Mr Hampton's paper earlier in this session should have provoked the two communications from Mr Preston to which we have just listened. It is satisfactory to learn that the treatment is so readily put in a form which is obviously consistent in its dimensions: on the practical and numerical side any serious error would have become evident at an early stage through a discrepancy between the magnitude of the actual and the anticipated residual strains. The chief interest to my mind, however, centres on the subject matter of the other paper—the justification of the Adams and Williamson law of decay of strain in glass at a fixed temperature. When I first saw Mr Hampton's paper, this law seemed to me of so peculiar a character that I looked up the account of its derivation in the *Journal of the Franklin Institute*.

It was at once apparent that the mathematical part of this investigation was meaningless. The observations were fitted to a law of the type

$$\log \frac{F_0}{F} = mt^b,$$

where $F$ is a measure of the strain, $t$ is the time, and $m$ and $b$ are unknown constants. The determination of these numbers was attempted by plotting the logarithm of $\log F$ against the logarithm of $t$. Both this procedure and the assumption of the above form for the equation are in general inadmissible, for the reason that the origin from which $t$ is measured is a vital consideration with an equation of this kind except in the special case $b = 1$. The simplest form of this type which may be assumed is

$$\log \frac{F_0}{F} = m\{(t + t_0)^b - t_0{}^b\}.$$

Adams and Williamson succeeded to their own satisfaction in fitting the observations to a law where $b$ ranged from 0·5 to 0·7 for different glasses; from what has already been said $b$ as thus determined is a function of a particular event—probably the strain when the first observation was made—and is not a physical constant of the glass. This equation, however, was superseded by another empirical equation which fitted the observations about equally well, for the reason that the new equation was more convenient for extrapolation. This is the equation which Mr Preston has justified by dimensional reasoning on the supposition that viscosity is the only physical property which enters into the problem. In reality other factors must be considered, and, not to transgress the bounds set by the papers before us, the conclusion I draw from Mr Preston's result is that the theory of dimensions so far shows that the rate of decay is directly proportional to the fluidity but may be any function of the strain. Other physical considerations than those of dimensions, however, may guide us, and on such grounds the law as given both by Mr Hampton and Mr Preston is open to objection. Taking Mr Preston's form

$$-\frac{df}{dt} = \gamma f^2,$$

it is to be observed that $\gamma$ is a physical constant of the material, which in any system of units is expressed by a positive number. Since $f^2$ is also necessarily positive, $df/dt$ must always be negative, whether $f$ is positive or negative. That is to say, if this equation is a true representation of the facts, a positive strain will tend to diminish in time to zero, but a negative strain will increase indefinitely as time goes on. I think it will be admitted at once that this consideration is fatal to the equations in the precise form in which they stand.

Objections of this and other kinds can be met, but in the long run we depend upon fitting observations to types of equation which are admissible on dimensional and other grounds. Numerical analysis of the actual observations is therefore important. My own conclusion after examining in this way the observations of Adams and Williamson is that the observations are good, but that, where a con-

siderable number of settings have been taken, departures from the law they adopt are indicated. In one special case, where prolonged observations at an unusually low temperature were made, this departure was specially noted by the authors. Both this and the departures in the annealing range itself suggest to my mind that the real law involves, not an ultimate state in which strain is absent, but the decay of the strain to a finite limiting value which will depend on the temperature. Such a distinction may not be of moment to the manufacturer of glass, but it is of such interest to the physicist that I very much hope extensive and accurate observations on the decay of strain in glass under various impressed conditions will be carried out in this country as well as elsewhere.

Mr W. M. Hampton: Although Mr Preston has one or two criticisms to make about my paper, it is comforting to know that his final conclusions are satisfactory.

Mr Smith's comments are quite as valuable as the paper itself and it must be admitted that the present form of the fundamental equations is not entirely satisfactory as at least two factors are not taken into account.

To find the closer approximation which Mr Smith requires appears to me to be beyond the range of our present experimental skill since temperature control of a small furnace at, say, 500° would be needed to an accuracy of a small fraction of a degree centigrade.

Mr Smith's deduction from the numerical data that the strain tends to reach a definite value which is not zero is a very interesting one and confirms Mr Twyman's* experimental results, although these are not entirely convincing, as the experiments are rather meagre and the individual errors rather large.

It appears that Adams and Williamson and myself were exceedingly fortunate in using an expression which, while not entirely covering the experimental facts, is nevertheless not inconsistent with the results of dimensional analysis.

Mr Preston (communicated): Mr Smith has hit two of the most interesting nails quite squarely on the head. I did not desire in my papers to cover any more ground than the subject matter of Mr Hampton's paper and Mr Smith's comments thereon warranted; but I have dealt with the real significance of the matter in a separate letter to Mr Hampton.

The dimensional equation $-df/dt = \gamma f^2$ is a particular case of a more general equation that cannot be deduced by dimensional reasoning alone, viz., $-d\mathcal{f}/dt = \gamma \mathcal{f}^2$, where $\mathcal{f}$ is the difference between the existing stress and the stress appropriate to the externally-impressed rate of flow. $\mathcal{f}$ is always positive, and $-d\mathcal{f}/dt$ is necessarily always negative, i.e. the stress always tends to move towards its appropriate value. It moves at a rate proportional to the square of the distance to go.

This law of course covers far more phenomena than the ordinary process of annealing, and it also demolishes entirely Clerk Maxwell's conceptions of the process of viscous flow. The law was discovered experimentally.

The general differential equations above given cannot be integrated unless we know that $\gamma$ is a true constant. This is Mr Smith's second nail. $\mathcal{f}$ in the annealing

* Twyman, *Journ. Soc. Glass Tech.* 7 (1923) 199.

of some substances is not $(f - 0)$ but $(f - f_s)$, where $f_s$ is the yield stress; the substance being a plastic not a viscid one. Glass may be such a substance: I am not equipped to find out and I hope Mr Hampton or Mr Smith is. My own experiments have related to various materials that can be handled at room temperature, and the general differential equation seems to be of very wide applicability.

There is a great deal of confusion of thought at the present time in discussing the viscosity of glass because it seems to be thought that the coefficient of viscosity or the modulus of elasticity is some eternal verity, something fixed, tangible, and real, an actual "physical constant" of the material. Actually these quantities are mathematical abstractions, defined as the ratio of stress to flow or of stress to stretch. There is no reason why they should be "constants" at all: it is simply observed that sometimes they are. Most recent experimenters have neglected to prove whether their materials have such "physical constants" or not: they just assume that the material possesses a modulus (say), and that the modulus is a constant at all stresses.

When the mathematical quantities are stripped of the physical reality with which they tend to become invested, the problem becomes a purely mathematical one, and the method of dimensional analysis is bound to give the correct answer. The question whether our mathematical fictions then correspond sufficiently well with physical facts is one that must be investigated experimentally. Thus in the present instance, Adams and Williamson's result may be taken as an experimental investigation as to whether $\gamma$ is a constant: and the answer may be interpreted "At high temperatures, yes; at low temperatures, not quite."

This is tending to become somewhat of a lecture, but what Mr Smith evidently wants is something more general than a few comments on annealing: in fact, an account of the phenomena of flow in viscous and plastic bodies. This subject is much wider in scope and correspondingly more interesting. I hope to deal with it one of these days.

# NOTE ON THE COSINE LAW

By T. SMITH, M.A., F.Inst.P.

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*MS. received, 16th April*, 1925. *Read and discussed, 14th May*, 1925.

*ABSTRACT.* Objections raised to the statement that rays selected by a cosine relation determine caustic surfaces in the object and image spaces, and to some points in the proof of the cosine law, are considered in some detail and the adoption of the original enunciation of the theorem justified as opposed to the modified form suggested by Hertzberger and approved by Boegehold. A direct derivation of the analytical form of the law of refraction from the cosine law is given.

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From a note in the third edition of Czapski's *Theorie der optischen Instrumente* I had learnt earlier that some observations on my paper "The Optical Cosine Law"* had been published in Germany by M. Hertzberger†, but it is only within the last few days that I have found his article. In this article objections are raised to my enunciation and proof of the law, and it is further suggested that assumptions have been made tacitly which ought to be justified. In presenting the law to his readers Hertzberger has introduced the amendments he considers necessary in the enunciation, and, while following the general lines of proof of the original paper, brings out clearly the deficiencies he has found. The same journal for 1924 contains two contributions on the subject from H. Boegehold‡, who concurs in Hertzberger's criticisms. Since the previous discussion has not been found entirely convincing by these two experienced opticians, it is not unlikely that similar difficulties have been found by others who may be more diffident in voicing their objections. A few remarks may therefore not inappropriately be offered on the specific difficulties to which attention has been called.

The earliest criticism made of the law—the proof then offered differed from that published in these *Transactions*—was to the effect that the law was only true for first order aberrations. That was in 1912, but neither then nor since have I been able to find the remotest justification for this statement. It is satisfactory to note that in the criticisms now emanating from Jena it is admitted that the law is undoubtedly true, irrespective of the order of the aberrations present, in large classes of cases—possibly including all those of practical importance—though it is claimed that as a perfectly general law it is not yet established. The cases in which it is admitted include those in which the rays in the object space diverge from a point, and from this it readily follows that it also holds when the rays touch any caustic

* T. Smith, *Trans. Opt. Soc.* 24 (1922–23) 31.
† M. Hertzberger, *Cent.-Ztg. f. Opt. u. Mech.* 44 (1923) 211.
‡ H. Boegehold, *ibid.* 45 (1924) 107, 295.

which can have a point image. The fundamental objection now raised, on which I think all the others depend, challenges the statement that the rays which satisfy the condition

$$\cos\theta = p\cos\theta' + q \qquad \ldots(1)$$

determine caustic surfaces in the object and image spaces. Hertzberger observes that the statement is undoubtedly incorrect, since the rays which satisfy this condition have three infinite degrees of freedom, while the rays associated with a caustic surface have only two degrees of freedom. On making the amendments in the enunciation of the law required to meet this objection, Hertzberger finds that the existing arguments are inadequate, particularly important being the addition of a proof that $q$ has the same value for all the rays of the three-dimensional manifold concerned.

Now Hertzberger's objection may at first seem to the reader to be conclusive, but I propose to show that it is a false inference drawn from the comparison of two ray aggregates, one general and the other special. It will be demonstrated that rays having three degrees of freedom determine a surface, that the theorem is correctly stated in its original form, and that with this restoration the deficiencies to which Hertzberger has called attention disappear.

The error in the argument on the number of degrees of freedom has perhaps escaped detection because we are so accustomed to regard space as a three-dimensional aggregate of points and a surface as a two-dimensional aggregate. When we deal with straight lines instead of points these numbers must be changed. A straight line is determined by any two points on it. Suppose then that we consider the points in which a straight line meets two parallel planes. Since each of these points has two degrees of freedom, it is obvious that rays filling space have four degrees of freedom. This also follows from the fact that if $l, m, n, \alpha, \beta, \gamma$ are the co-ordinates of any line, the six are connected by the two relations

$$l^2 + m^2 + n^2 = 1,$$
$$l\alpha + m\beta + n\gamma = 0.$$

It should be obvious from this that the imposition of a single constraint, which reduces the degrees of freedom from four to three, involves the definition of a surface. In the actual problem the condition is presented in a form which involves the direction cosines of both the incident and the refracted ray, and the consequences of that constraint may be derived directly from that form.

As before, let $\mathscr{E}$ be the eikonal, regarded as a function of $M$, $N$, $M'$, and $N'$. If $x, y, z$ are the co-ordinates of any point on the incident ray,

$$\mu\left(y - \frac{M}{L}x\right) = \frac{\partial\mathscr{E}}{\partial M}, \quad \mu\left(z - \frac{N}{L}x\right) = \frac{\partial\mathscr{E}}{\partial N} \qquad \ldots(2).$$

These two equations may be solved for $M'$ and $N'$, and the resulting values substituted in the condition (1), which when written at length takes the form

$$Ll + Mm + Nn = p\,(L'l' + M'm' + N'n') + q.$$

The object space relation derived in this way is of the form

$$F\left(M, N, y - \frac{M}{L}x, z - \frac{N}{L}x\right) = 0 \qquad \ldots(3)$$

and connects the four line co-ordinates of every incident ray included in the group defined by (1). Through an ordinary point of the object space it is apparent that a singly-infinite sheaf of these rays passes, but at particular points the aggregate of rays may be of the second order. Let $(x, y, z)$ be such a point: this point is then common to the ray $L, M, N$ and the neighbouring rays found by varying $M$ and $N$ independently. That is to say, at this point the conditions

$$\frac{\partial F}{\partial M} = \frac{\partial F}{\partial N} = 0 \qquad \ldots(4)$$

are satisfied, where $x$, $y$, and $z$ are regarded as constants during the differentiations. By eliminating $M$ and $N$ between the three relations (3) and (4), an equation

$$\phi(x, y, z) = 0 \qquad \ldots\ldots(5)$$

is obtained for the locus of points in the object space at which the ray union is of a higher order than elsewhere, and this locus is obviously in general a surface. In a similar way the equation of the surface in the image space where the ray union is of a higher order may be determined. The relation (1) therefore determines two surfaces, one in the object space and one in the image space, where the ray union is of higher order than elsewhere, and it is precisely to surfaces having this property that the word caustic is applicable. This is sufficient to justify the reference to caustic surfaces in the enunciation, and also the use of known properties of rays and surfaces in the proof. The meaning and constancy of $q$ are at once obvious, as may be seen from the proof of the converse theorem where the relation is obtained in the form

$$\mu\sigma \cos\theta - \mu'\sigma' \cos\theta' + \epsilon = 0,$$

$\epsilon$ being the increment in the optical path between corresponding parts of the caustic surfaces due to the small displacements $\sigma$ and $\sigma'$.

Although rays with three degrees of freedom determine caustic surfaces, the difficulty remains that in the familiar cases of practical optics the rays associated with a caustic have but two degrees of freedom. The explanation of the discrepancy is to be sought in the very general character of our law. The group of rays selected by the condition (1) is great compared with the bundle concerned in imaging a single object point, but since the property in which we are interested has been proved for every member of the larger aggregate, it is necessarily true of any smaller group contained in this aggregate. The reason why we are ordinarily interested in the smaller group is because it is only within these narrower bounds that permanent phase relationships subsist with realizable sources of light. In consequence of this limitation—which from our present point of view is accidental—the density of the rays in which we are interested is everywhere lowered; through most points of the object and image spaces only a single ray passes, and at the

caustic surfaces the order of the ray union is reduced from two to one. That the surface which we have called the caustic surface in the three-dimensional aggregate is also the caustic surface when the order is lowered follows from an enumeration of the points contained in the surface. That is to say we are maintaining consistency in the usage of the word caustic.

In the instances that we have to consider in applied optics the rays under consideration obey two conditions of the cosine form. In the object space they therefore determine two caustic surfaces, and similarly two surfaces are involved in the image space. In general we cannot go beyond this stage, as is illustrated in Boegehold's second article referred to above, where he shows that if three independent conditions of this form are satisfied they are reducible to

$$L' - L = M' - M = N' - N = 0,$$

which by the general theorem implies the magnification $\mu/\mu'$ in all directions.

The question has been raised whether the ordinary sine law is derivable from the cosine law without the introduction of other considerations. The following derivation of the analytical form of the law of refraction, involving both its conditions, seems to meet the question. Obviously the conditions to be satisfied are that the displacements of the object and image rays are to be identical in magnitude and direction so long as the infinitesimal displacements are in the image surface, the separation of object and image caustics being always zero. Thus if $l$, $m$, $n$ is any direction in the refracting surface at the point of refraction, we must have

$$(\mu'L' - \mu L)\, l + (\mu'M' - \mu M)\, m + (\mu'N' - \mu N)\, n = 0$$

as a consequence of

$$fl + gm + hn = 0,$$

where $f$, $g$, $h$ are the direction cosines of the normal to the surface at the point of refraction. In other words we must have

$$\frac{\mu'L' - \mu L}{f} = \frac{\mu'M' - \mu M}{g} = \frac{\mu'N' - \mu N}{h},$$

which is the ordinary expression for the law of refraction.

One word more ought perhaps to be said in conclusion. The mention by Boegehold in the title of his first article of Conrady's cosine condition led me to wonder whether I had inadvertently failed to notice previous work on this subject. The position, however, appears to be that, apart from simplifications connected with the special problem (the Abbe sine law) under discussion, Conrady's formula (which is not formally stated as a cosine relation) gives the phase variations for a linear group of rays without attention to ray positions after refraction, while in the case of the cosine law the displacements are always consistent with the law of refraction and hence also with the law of stationary path. This distinction is closely interconnected with the difference in the order of the groups of rays involved in the two cases.

## DISCUSSION

Dr H. Boegehold (communicated): A. E. Conrady confined his method to axially symmetrical cases: he did not put his condition in the cosine form. To the former limitation I have referred repeatedly. Both limitations can easily be removed, and as a result, in several places in Czapski's textbook I was able in the years 1921 and 1922 to apply the cosine condition without knowing Mr Smith's law, to which I could refer only briefly and by way of supplement (pp. xii, 233, 280, 284). There is no doubt that Mr Smith was the first to enunciate the law in the cosine form and that in a very general manner; my reference to Conrady was probably due to my recollection of the assistance which I have continually received from the Conrady condition.

Mr Smith remarks that an important difference exists between the order of the ray group for which the law is enunciated and that of the group to which it is applied. The application to the ordinary sine condition may be set out as follows:

With an axially symmetrical arrangement the equation $n \sin u = n'\beta \sin u'$ defines in general a manifold $M_3$ of the third order, and in the image space a similar manifold $M_3'$. The fulfilment of the sine condition involves the following:

(*a*) to $M_3$ belongs the manifold $M_2$ of the second order comprising all rays which pass through an axial point $P$ in the object space;

(*b*) to $M_3'$ belongs the manifold $M_2'$ of the second order comprising all rays which pass through an axial point $P'$ in the image space;

(*c*) the manifolds $M_3$ and $M_3'$ correspond.

The condition (*c*) is not immediately deducible from (*a*) and (*b*). If now Mr Smith's law holds, there corresponds to a congruent manifold $\overline{M_3}$, displaced perpendicularly to the axis, a displaced manifold $\overline{M_3'}$; both include manifolds $\overline{M_2}$ and $\overline{M_2'}$ of the second order which pass through the points $P_1$ and $P_1'$, but it requires a special proof to show that $\overline{M_2}$ and $\overline{M_2'}$ correspond to one another and that $\overline{M_2}$ is not conjugate to some other manifold $\overline{M_2''}$ which need merely be contained in $\overline{M_3'}$.

The case will be similar in other applications to doubly-extended manifolds.

It was not my intention to contest that the sine condition can be derived indirectly from Mr Smith's law.

There appears to be a further curious exception to Mr Smith's law in the case where in the object and image spaces the triple manifold consists of all straight lines which cut a particular straight line (the axis in axially symmetrical systems), and a displacement in the direction of this line is assumed.

Mr Smith (communicated): I am glad to have this opportunity of thanking Dr Boegehold for the interesting contributions he has made to the literature of the sine and the allied conditions. I am specially interested to learn that he had independently made extensive generalizations of the sine condition on the lines employed by Conrady.

In the particular applications of the cosine law to which Dr Boegehold has

referred, I fancy that any distinctions we make depend upon the particular points of view from which we see the subject rather than upon anything involved in the law itself. In the way in which I view it, the law as it stands is sufficient to deal with these special problems, and the difficulty of selecting the correct twofold complex of rays from the displaced threefold aggregate does not arise. By stating that the system is axially symmetrical a limiting condition for the rays has been given, which in conjunction with another condition leaves a manifold of rays of only two degrees of freedom. Thus the entire manifolds in such cases degenerate to the second order, and the problem of selecting a second order manifold from a third order manifold does not arise. Alternatively we may regard the problem as one in which two conditions of the cosine form are given, the condition

$$n \sin u = n'\beta \sin u'$$

being equivalent to

$$n \sin u \cos \delta = n'\beta \sin u' \cos \delta',$$

$$n \sin u \sin \delta = n'\beta \sin u' \sin \delta',$$

when $\delta = \delta'$.

The problem of the rays meeting lines in the object and image spaces parallel to the direction of displacement from the present point of view similarly involves no ambiguity, but as it involves a somewhat lengthy discussion to treat it fully it is not very conveniently dealt with in a short reply.

# NOTE ON THE STAEBLE AND LIKOTZKY CONDITION

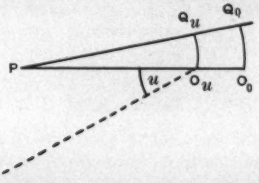
By Dr H. BOEGEHOLD

(Messrs Carl Zeiss, Jena.)

[Communicated by Mr T. Smith, M.A.]

*MS. received, 25th June, 1925.*

I would like to draw attention to the fact that the cosine law can be proved for and applied to more general cases than a parallel displacement, at least in the special case of normal bundles. Instead of the proof I give the application to the generalization of the sine condition by Staeble and Likotzky (cf. Czapski, p. 253). Let $PO_uO_0$ in the accompanying figure be the axis in the object space and consider




the axially symmetrical bundle of rays which passes through $O_0$ for infinitely small inclinations and through $O_u$ for the inclination $u$. To this there corresponds in the image space an axially symmetrical bundle where accented letters have the corresponding meanings. If a rotation through an infinitely small angle is made in the object space about an axial point $P$, the inclined axis $PQ_uQ_0$ takes the place of $PO_uO_0$. The condition for isoplanatic correction is that in the image space the same thing happens for an inclined axis $P'Q_u'Q_0'$. If now $AO_u$ is any given ray, and $A'O_u'$ the corresponding one ($AO_u$ and $A'O_u'$ in general do not lie in the plane of the diagram), the cosine condition

$$n'O_u'Q_u' \cos A'O_u'Q_u' - nO_uQ_u \cos AO_uQ_u = \text{const.} \qquad \ldots\ldots(1)$$

follows, in which, however, $O_u'Q_u'$ and $O_uQ_u$ are not constant. Now

$$\cos AO_uQ_u = \sin u \cos \delta,$$

where $u$ is the angle of inclination, $\delta$ the azimuth of the plane $AO_uP$. Since $\delta = \delta'$ and the equation must also hold for $u = 0$, we have

$$n'O_u'Q_u' \sin u' - nO_uQ_u \sin u = 0$$

or

$$n'O_0'Q_0' \frac{P'O_u'}{P'O_0'} \sin u' - nO_0Q_0 \frac{PO_u}{PO_0} \sin u = 0 \qquad \ldots\ldots(2).$$

This is the Staeble condition (cf. Czapski, p. 256, 15 *b*).

## DISCUSSION

Mr T. Smith (communicated): I am interested both in Dr Boegehold's present note and in the more general theorem to which he refers. The work of Staeble and of Likotzky was previously quite unknown to me. After I had established the cosine theorem I naturally looked to see whether an equally general result held for rotational instead of rectangular displacements. It became at once evident that rotational displacements were in general on a quite different footing from those for which the cosine law holds, and I formed the opinion that the relations derived from the condition in their most general form were of less importance. It does not of course follow that in special cases interesting deductions may not be possible, such as that reached in the present note.

# THE OPTICAL PROPERTIES OF AMETHYST QUARTZ

By Professor C. V. RAMAN, F.R.S., and K. BANERJI

(The University, Calcutta.)

*MS. received, 7th May, 1925. Read, 11th June, 1925.*

*ABSTRACT.* Circularly-polarized waves of light, in passing through a section-plate of amethyst quartz, would obviously be retarded unequally in the alternate laminae having right- and left-handed optical activities. Diffraction effects must therefore arise, and these can actually be observed even in almost colourless varieties of the quartz. The section-plate behaves in effect as a phase-change diffraction grating, and the question of the visibility of its structure with and without the aid of polarizers and analysers is discussed in the light of Abbe's theory of microscopic vision.

## Introduction

The periodic alternation of layers rotating the plane of polarization in opposite directions, so beautifully shown by amethyst quartz when a section-plate cut normal to the optic axis is viewed between nicols, has often been observed and described. Some fine illustrations of the type of sectorial repeated twinning characteristic of amethyst quartz are reproduced as Plates XXV and XXVI of Tutton's new and very attractively written book, *Natural History of Crystals*, in which several pages are devoted to a description of amethyst, and his account will no doubt stimulate interest and inspire fresh studies of the properties of this remarkable substance. The view now generally accepted is that the twinning is of the simple Brazilian type in which the plane of twinning is parallel to a pair of faces of the second-order hexagonal prism, the actual surfaces of junction of oppositely active parts being, however, very varied in their distribution and character. The effects observed in the polarization microscope are explained in a simple way as due to the superposition of the opposite rotations of the plane of polarization produced by the two types of quartz. Parts of the section-plate in which equal thicknesses of the oppositely active parts occur would appear quite black between crossed nicols, other regions exhibiting illumination and colour varying with the thickness of the two kinds traversed by the light. The effect of rotating either the polarizer or the analyser on the structure seen is similarly elucidated.

We wish to draw attention to another class of optical effects arising from the structure of amethyst, and to suggest a new view-point from which to consider the phenomena observed in the polarization microscope with this substance.

### Diffraction Effects

Consider the case of a section-plate cut normal to the optic axis, the alternate strips of oppositely active parts being assumed to be of equal width and separated by planes normal to the plate. If the incident waves of light be unpolarized, they may be resolved into two wave-fronts circularly polarized in opposite senses but having no determinate phase-relation which may therefore be considered separately. An incident circularly-polarized wave, assumed to be initially plane, traverses the alternate strips with different velocities, and on emergence from the plate must therefore become corrugated. The corrugated wave-front should, according to well-known principles*, give rise to diffraction effects and should in fact, at a sufficient distance from the crystal, resolve into sets of plane waves corresponding to a central undeviated pencil and diffraction spectra of the first, second, and higher orders. The same result should occur when the boundaries of separation between the twins are not plane or parallel to the optic axis, or if the alternate strips are not equal in width, the distribution of intensity in the diffraction spectra being alone influenced by these factors.

That amethyst does give rise to diffraction effects as indicated by theory is easily verified by observation. For this purpose the polarizing and analysing nicols are not necessary and may be removed. A narrow source, e.g., a "Pointolite" lamp, should be used, and the light so adjusted that it passes normally through the plate. Coloured diffraction fringes, arranged periodically and running parallel to the lines of the structure, are then seen in the field behind the plate. The fringes (which are of the Fresnel type) are shown just as conspicuously by nearly colourless varieties of amethyst as by the more deeply tinted ones, and this circumstance, together with the fact that they disappear completely when the viewing microscope or eyepiece is focussed on the plate, proves clearly that the diffraction effect is due to the periodic change of phase produced by the structure and not to any periodic variation of transparency. The diffraction spectra of the Fraunhofer type due to the structure may also be seen when the observing eyepiece is focussed on the image of the source of the light formed by the objective.

### Amethyst as Phase-Change Grating

Since a section-plate of amethyst quartz is thus in effect a phase-change diffraction grating, its appearance in the polariscope may be interpreted in the light of the Abbe theory of microscopic vision, and in fact furnishes very convenient illustrations of that theory. The first point worthy of remark is that little or no structure is observable when the polarizer and analyser (either or both) are absent, though, as we have seen, they are not required to enable the structure to give rise to diffraction effects. This has a bearing on the general question of the visibility of phase-change structures. Lummer and Rieche† have discussed the

* Rayleigh, "On the Dynamical Theory of Gratings," *Scientific Papers*, vol. v, p. 388.

† Lummer and Rieche, *Bildentstehung im Mikroskop*, F. Vieweg und Sohn (1910), pp. 74–76.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

case of a *sudden* change of phase, and found that the boundary at which this occurs should be visible as a distinct minimum of intensity. This is borne out by observations with structures having sharply-defined laminar boundaries*, but when the boundaries are less sharp, their visibility is greatly prejudiced and becomes practically zero. In the present case, the phase-changes arise in traversing a considerable thickness of the crystal, and even if the boundaries between the oppositely active parts of the crystal were perfectly plane and normal to the surface of the section, the wave-front on emergence from the crystal cannot present any *discontinuous* changes of phase over its area. The failure to observe any marked structures in the absence of the polarizer or analyser is thus intelligible. When the polarizer and analyser are put in, the structure ceases to be merely a phase-change grating and becomes one in which there is also a periodicity of effective transparency, and an enormous increase in the visibility of the structure is the result. An improvement in the visibility of the structure of twinned crystals of potassium chlorate when polarized light is used has been noted by Lord Rayleigh†.

## Visibility of Structure

The matter may also be regarded in another way. When the incident light is unpolarized, it may be resolved into two components circularly polarized in opposite senses and having no determinate phase-relations, and the diffraction spectra given by them are merely superposed. If, however, the incident light be polarized, the two sets of circularly-polarized diffraction spectra are in a definite relation of phase which is different from that of the central undeviated pencils, since the accelerated parts of one wave correspond to the retarded parts of the other and vice versa. Hence, the introduction of an analyser enables the relative intensity of the central image and of the diffraction spectra to be varied, and therefore, in accordance with the Abbe theory, also the appearance of the structure under the microscope.

The four pictures reproduced in the plate are photographs in the polarization microscope of a practically colourless piece of amethyst quartz 2·5 mm. thick. They were taken with analyser and polarizer in different relative positions, and using monochromatic light of different wave-lengths. The differences in the appearance under the varying conditions are sufficiently striking. In Fig. 1, the nicols were crossed and green light was used. In Fig. 2, the nicols were parallel and violet light was used. Figs. 3 and 4 correspond to intermediate positions of the nicols, Fig. 3 being obtained with violet and Fig. 4 with green light.

* N. K. Sur, *Proc. Ind. Assn. Science*, 7 (1922) 126–144.

† Rayleigh, *Proc. Roy. Soc.* A. 102 (1923) 669–670

# ABSTRACTS OF PATENT SPECIFICATIONS

227456*. Soc. Anon. Industriale San Giorgio. RANGEFINDERS.

Short-base rangefinders are provided with a pair of prisms between one of the objectives and the eyepiece. A tilting movement of the instrument produces rotation of the prisms in opposite directions, and gives a direct reading of the altitude of the target. Coincidence of the partial images is secured by a longitudinal movement of the prisms.

228125. Bignold, E. J. VIEWING APPARATUS.

An appliance for use in nature study comprises a table upon which small objects may be mounted, and which is supported so as to rotate beneath a focussing magnifier. The lower part of the casing furnishes a magazine fitted to receive additional mounts.

228207. Crow, A. B., and another. OPTICAL SYSTEMS.

Optical systems for photographic or projection purposes, especially for colour work, include two or more achromatic prisms, placed in the position of minimum deviation for rays parallel to the optical axis of the lens system, with their thinner edges meeting at the axis. The arrangement is designed to produce identical images, or enable a number of images to be projected in superposition, appropriate filters being employed if necessary.

228354. Bracey, R. J., and another. MAGNIFIERS.

A magnifying lens is so constructed as to provide for the illumination of the object under observation. The surface of contact of the components of a positive combination may be silvered except for an aperture in the centre; or a meniscus lens of suitable curvature may be employed. In each case, the focal lengths of the refracting and the reflecting system are approximately equal.

228366. Lilienfeld, J. E. VEHICLE LAMPS.

The bulb itself is made lens-shaped, with a back parabolic portion which is silvered on the inside, and preferably provided with a protecting backing. A conical reflector throws the light back upon the main mirror surface. The filament, which may be conical, is located partly behind, but mainly in front of, the focus.

228640. Aldis, H. L. SIGHTING-TELESCOPES.

Telescopic sighting-apparatus is fitted with a reversed Galilean system, in order to obtain a wide angular field. The resulting reduction in size may be partially corrected by providing a weak positive lens in the eyepiece. A pair of pivoted prisms enable objects directly ahead and astern to be brought into view simultaneously with objects immediately below.

228704. Skennerton, W. I. CONDENSERS.

Two bi-convex lenses are employed, having a converging meniscus placed between them. The lenses are separated by a series of spacing-rings, of which those near the illuminant may be filled with asbestos.

228904*. Cassan, I. VEHICLE LAMPS.

In front of the filament there is placed a spherical or coned reflector, to intercept the direct beam and throw it back upon the main mirror. This may be contained within the bulb, or be formed by silvering a portion of the glass of appropriate form.

* Specification published before acceptance.

229024. Pilkington Bros., and another. GRINDING AND POLISHING GLASS.

Flat glass plates are carried beneath two series of runners, one set of which effects the grinding and the other the polishing. The runners are given a differential speed relative to the plates, which may be traversed two or more times. In a modified apparatus, the runners travel at the same speed as the glass, but are given a curved motion, mainly transverse to the general direction of traverse.

229028. Ireland, F. VEHICLE LAMPS.

The optical system of an anti-dazzle lamp consists of a reflector, preferably parabolic, axially adjustable within the casing of the lamp, and two preferably plano-convex lenses carried by a telescopic mount forming the front of the lamp. The bulb is also adjustable in position.

229039. Engholm, F. W., and others. REFLECTORS.

For projecting light or other forms of radiation, the source is placed at the common focus of two opposed reflectors, both generated by conic sections. Two parabolic mirrors produce a parallel beam, which may emerge through an aperture in the front reflector. If the front mirror is plane, the beam is convergent. A divergent beam is obtained by using an elliptical reflector, having the source of light placed at one focus, an aperture being made at the opposite end of the major axis.

229169. Nolan, T. A. OPTICAL ADVERTISING.

A projection lantern is employed in connexion with an endless film, upon which the device to be displayed is imprinted along a spiral line, the arrangement being such that as the film progresses, it is fed transversely, so that the device is always maintained in the proper position in the sight aperture.

229201. Lebby, S. L. SIGNAL LAMPS.

The bulb is mounted upon a short pillar which admits of universal adjustment, and is backed by a lens mirror. The front glass is carried by a door, the inner face of which is painted a dull colour to provide a focussing-target around a central aperture of a size corresponding to the proper width of the beam at that point.

229287*. Fagerholm, P. E. THEODOLITES.

A viewing-system is fitted to project an image of the horizontal circle into the field of the telescope. A tube projecting towards the circle, and provided with a lens, is carried round to follow the rotational movement of the telescope. A prism mounted upon the trunnion axis, but moving at half speed, directs the light into the eyepiece, and keeps the image steady as the telescope is tilted.

229604. Lehwiss, E. E. PROJECTION APPARATUS.

Stationary or animated images are projected from opaque films, which are illuminated by a number of sources placed symmetrically with respect to the object. The surface of the film is glazed, so that the light falling upon the black parts of the object is regularly reflected and does not enter the objective. In the system described, two lamps are utilized, each backed by a reflector arranged to produce an image of the filaments between the filaments themselves. Two condensers are fitted in each system, one near the source and one adjacent to the object.

230500*. Burkhard, E. P. OPTICAL APPARATUS.

A tube provided with a magnifying lens or kaleidoscope mirrors is used for viewing small pictures, which are formed upon a film carried by a frame mounted upon the end of the tube. The film rollers are rotated against the friction of check springs.

R. J. T.

* Specification published before acceptance.

moved from the Chair and carried unanimously, due notice having been given to all members:

(1) That the existing Rule VI be replaced by the following:

*Rule VI.* Student Members. The class of Student Members is restricted to

(*a*) persons under twenty-one years of age,

(*b*) persons attending a full time course of instruction in science at a recognized institution.

A Student Member on ceasing to be qualified as above, if desiring to remain a Member of the Society, must apply for transference to the roll of Members, and such transference shall be made by the Council without payment of entrance fee.

(2) That Rule VIII be amended by substituting, for "£1. 1*s*. 0*d*." in line 12 of said Rule, "10*s*. 6*d*."

*11th December*, 1924

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., Vice-President, in the Chair.

Messrs T. F. Connolly and W. H. A. Fincham were elected Hon. Auditors for the Society's accounts for the year 1924.

The following papers were read:

"The Transformation of Trichromatic Mixture Data: Algebraic Methods." By J. GUILD, A.R.C.Sc., F.Inst.P., F.R.A.S. (Read in abstract by Mr T. SMITH.)

"An Equipment for Visual Spectrophotometry." By J. GUILD, A.R.C.Sc., F.Inst.P., F.R.A.S. (Read in title only.)

Dr L. C. MARTIN exhibited a Simple Microphotometer.

Professor A. F. C. POLLARD demonstrated a Telescope Support constructed by Professor C. V. BOYS.

*7th and 8th January*, 1925

Fifteenth Annual Exhibition of Electrical, Optical and other Physical Apparatus held at the Imperial College of Science, jointly with the Physical Society of London.

The following discourses were given:

"Some Experiments with Interferometers." By F. TWYMAN, F.R.S.

"Talking Motion Pictures." By C. F. ELWELL, B.A., M.I.E.E.

*15th January*, 1925

Meeting held at the Imperial College of Science.

Professor F. J. CHESHIRE, C.B.E., in the Chair.

The following paper was read:

"A Peep into Herschel's Workshop." By Dr W. H. STEAVENSON, F.R.A.S.

Dr STEAVENSON presented to the Society a Catalogue of Instruments made and (or) used by Sir William Herschel, as preserved at Slough and examined there in 1924, May and June.

Dr P. P. SCHILOVSKY gave a demonstration of Slow Speed Precision Gear governed from a Distance.

*12th February*, 1925

Annual General Meeting held at the Imperial College of Science.

Emeritus Professor A. BARR, F.R.S., President, in the Chair.

The Hon. Secretary presented the Annual Report* of the Council, which was approved.

The Treasurer's Report† and Balance Sheet‡ were presented by the Hon. Treasurer, and were approved.

The Chairman read the list of persons nominated by the Council for office in the year 1925–26 and declared all these duly elected as no other nominations had been received.

*President:* Mr T. SMITH.
*Vice-Presidents:* Sir FRANK DYSON, Instr.-Comdr. T. Y. BAKER, Emeritus Prof. ARCHIBALD BARR.
*Business Secretary:* Prof. A. F. C. POLLARD.
*Papers Secretary:* Mr F. F. S. BRYSON.
*Treasurer:* Major E. O. HENRICI.
*Librarian:* Mr J. H. SUTCLIFFE.
*Editor:* Dr JOHN S. ANDERSON.

*Council:*

| Already serving: | Elected, February, 1925: |
|---|---|
| Mr W. M. BRETT. | Mr D. BAXANDALL. |
| Prof. F. J. CHESHIRE. | Dr R. S. CLAY. |
| Mr W. B. COUTTS. | Mr E. F. FINCHAM. |
| Mr A. H. EMERSON. | Mr J. W. PERRY. |
| Mr H. H. EMSLEY. | Mr H. C. RAXWORTHY. |
| Mr P. F. EVERITT. | Mr J. RHEINBERG. |
| Mr J. GUILD. | |
| Mrs CHRISTOPHER LOCK. | |
| Mr F. C. WATTS. | |

Votes of thanks to the retiring Officers and Members of Council, to the Hon. Auditors, and to the Governors of the Imperial College, with Prof. Callendar, for the use of the College premises for the meetings of the Society were unanimously passed.

*12th February*, 1925

Meeting held at the Imperial College of Science.

Emeritus Professor A. BARR, F.R.S., President, in the Chair.

Professor BARR delivered his Presidential Address.

*12th March*, 1925

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

The following papers were read:

"Remarks on Mr Connolly's paper 'New Types of Levelling Instruments using Reversible Bubbles.'" By Dr A. FENNEL. (Read by Major E. O. HENRICI.)

"Additions to Our Knowledge of Old Spectacles: A Summary of Papers published

* See p. ix. † See p. xi. ‡ See p. xii.

in 1923–24 relating to the Subject of the Thomas Young Oration of 1923." By Professor MORITZ VON ROHR, Ph.D., Hon. M.D. (Read by Mr F. F. S. BRYSON.)

Mr E. F. FINCHAM exhibited Photomicrographs of Sections of the Human Eye.

Mr OGILVY described an exhibit of microscope accessories by Messrs Ogilvy and Co.

A letter from the Royal Microscopical Society was read inviting members of the Society to a Conference to be held at Sheffield during the week commencing 20th April.

It was announced that the Society of Glass Technology had extended an invitation to members of the Society to attend a General Symposium on "The Constitution of Glass" which would be held in London on 20th May.

*16th April*, 1925

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

The following papers were read:

"The Geometrical Solution of Colour Mixture Problems." By J. GUILD, A.R.C.Sc., F.Inst.P., F.R.A.S.

"The Double Square: A New Optical Appliance based upon the Optical Square." By J. W. GORDON, K.C.

Mr VAN NECK exhibited the "Aristol" Mirror Arc Projection Lamp and the Kahn-Goerz Workshop Microscope.

*14th May*, 1925

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

The following papers were read:

"The Fundamental Law of Annealing." By F. W. PRESTON. (Read by Mr W. M. HAMPTON.)

"The Dimensional Accuracy of Mr Hampton's paper on 'The Annealing of Glass.'" By F. W. PRESTON. (Read by Mr W. M. HAMPTON.)

"Note on the Cosine Law." By T. SMITH, M.A., F.Inst.P.

Mr WATSON BAKER exhibited some models of the Greenough Microscope with special attachments by Messrs W. Watson and Sons, Ltd.

*11th June*, 1925

Extraordinary Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

The following resolutions were moved from the Chair and carried unanimously, due notice having been given to all members:

(1) That the annual subscriptions of Fellows and Members be reduced to £2. 2*s*. 0*d*. and £1. 1*s*. 0*d*. respectively and that the entrance fee for all classes of membership be abolished.

(2) That these changes in the contributions of Members to the Society shall take effect on and after 1st January, 1926.

*11th June*, 1925

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

The President welcomed to the meeting Professor J. P. C. Southall, a past President of the Optical Society of America. Professor Southall thanked the President and the Society and expressed his pleasure at being present at a meeting of the Society, the work of which, he said, was greatly valued in America.

The following papers were read:

"The Changes in the Form of the Crystalline Lens in Accommodation." By E. F. FINCHAM.

"The Optical Properties of Amethyst Quartz." By Professor C. V. RAMAN, F.R.S. and K. BANERJI. (Read in title only.)

Messrs The Thermal Syndicate, Ltd., exhibited samples of transparent fused silica suitable for optical, illumination, and other purposes.

Dr R. S. CLAY and Mr T. H. COURT exhibited a Lucernal Microscope.

# THE OPTICAL SOCIETY

## REPORT OF THE COUNCIL TO THE ANNUAL MEETING OF MEMBERS, 12TH FEBRUARY, 1925

THE COUNCIL of the Optical Society presents the following Report for the year ending 31st December, 1924:

Nine Ordinary Meetings of the Society took place during the year, which were held at the Imperial College of Science and Technology by permission of the Governors and Professor Callendar, to whom the thanks and appreciation of the Society are due. In addition to the Ordinary Meetings, an Extraordinary General Meeting was held on 11th December when the existing rules were modified in favour of the admission and annual subscription of Student Members.

The replicas of the two Galileo telescopes at Florence and the Troughton Dividing Engine were the subjects of two papers delivered by Mr David Baxandall in January and February, and the thanks of the Society are due to Colonel H. G. Lyons, F.R.S., for permission to exhibit these two pieces of historical apparatus as well as to Messrs Cooke, Troughton and Simms, Ltd., for the loan of their Dividing Engine.

During the early part of the year the National Illumination Committee of Great Britain invited criticism and suggestions regarding a list of symbols of photometric quantities which was to be presented at a meeting of the International Commission on Illumination in July for international agreement. A report upon the matter was made by a sub-committee appointed by the Council and forwarded to the National Illumination Committee in February.

In conjunction with the Physical Society of London, the Fourteenth Annual Exhibition of Electrical, Optical and other Physical Apparatus was held at the Imperial College of Science and Technology on 2nd and 3rd of January and was largely attended as in previous years.

The Jubilee Celebrations of the Physical Society were held during March, when Congratulatory Addresses were presented by the representatives of many British and Foreign learned Societies. The Congratulatory Address of the Optical Society was presented by the President.

On this occasion the Physical Society extended a much appreciated invitation to Fellows and Members of the Optical Society, who had the privilege of examining a unique exhibition of instruments and apparatus used in historical researches in the Physical Sciences.

Throughout the year Fellows and Members were invited by the Board of the Institute of Physics to the series of lectures on "Physics in Industry" and the thanks of the Society are due to the Board for their courtesy.

The thanks of the Society are also due to the Illuminating Engineering Society who, in addition to a general invitation to all their meetings, specially invited members to a discussion on "Some Aspects of Railway Lighting" which was held during February.

A general invitation was also received from the Council of the Royal Photographic Society to the lecture delivered on 4th March by Professor Moritz von

Rohr, entitled "Contributions to the History of the Photographic Objective in England and America, 1800–1875," which was much appreciated.

At the celebrations of the Centenary of the Birth of Lord Kelvin which were held on 10th and 11th July the President represented the Society on the Committee of Honour.

The President also represented the Society at the International Mathematical Congress convened by the University of Toronto and the Royal Canadian Institute and held at Toronto from the 11th to the 16th of August.

During 17th, 18th, and 19th September the President represented the Society at the Centenary Celebrations of the Franklin Institute at Philadelphia.

On 9th July the conference of the Decimal Association was held, when Mr F. F. S. Bryson represented the Society.

As in the past, Mr R. S. Whipple has continued to represent the Society on the National Committee for Physics, while Professor F. J. Cheshire has represented the Society on the Board of the Institute of Physics.

In response to an appeal, a nearly complete set of the Transactions has been sent to the Tokyo Imperial University Library British Academy Committee, to assist in repairing the immense losses which the Tokyo Imperial University Library has sustained in the recent calamitous earthquake.

By reference to the Hon. Treasurer's Report it will be seen that the financial status is so satisfactory that a considerable surplus has accumulated. The Council, realizing the necessity of utilizing this surplus for the purpose of extending the activities of the Society, appointed a sub-committee to consider the matter. The Report of the sub-committee has been received and while one recommendation has resulted in a reduction of the annual subscription of Student Members, the other recommendations are being dealt with by a special sub-committee which will report during the early part of 1925.

## LIBRARY

The following publications have been received during 1924:

*Report on Radiation and the Quantum Theory* (Second Edition). By J. H. JEANS.
*Ophthalmo-Optical Manual*. By WILLIAM SWAINE, B.Sc.
*Grundzüge der Theorie der optischen Instrumente nach Abbe* (Third Edition). By S. CZAPSKI and O. EPPENSTEIN.
*Factory Costing*. By H. H. EMSLEY, B.Sc. (Eng.).
*The Structure of Crystals*. By RALPH W. G. WYCKOFF.
*Helmholtz's Treatise on Physiological Optics*. Translated from the Third German Edition. Vol. 1. Edited by Prof. JAMES P. C. SOUTHALL.
*Optical Measuring Instruments*. By L. C. MARTIN, F.R.A.S., D.Sc., A.R.C.S., D.I.C.
*Penrose's Annual: The Process Year Book*, Vol. 27. Edited by WILLIAM GAMBLE, F.R.P.S., F.O.S.
*Annual Report of the Smithsonian Institution*, 1921.
*Eastman Kodak Co., Abridged Scientific Publications*.
*National Physical Laboratory Reports for* 1922 *and* 1923.

*Nela Research Laboratory, Abstract Bulletin.*
*Scientific Papers of the Bureau of Standards* (*Optical*).

The *Japanese Journal of Physics* has been added to the Journals now regularly received, the list of which is given in the Report of the Council for the year ending 31st December, 1922.

The thanks of the Society are due to Professor F. J. Cheshire for his appreciated gift of an unbound set of the *Proceedings of the Royal Microscopical Society* from Volume 1, 1878, to the Volume for 1909.

### MEMBERSHIP

During the year eleven new Members were elected, of whom three are Corporate Members. Seven names were removed from the list under Rule IX, eighteen resignations were tendered, two applications became void under Rule III, and two members died. There is, therefore, a decrease of eighteen in the membership of the Society, leaving the number of members on 1st January, 1925 as follows:

| | |
|---|---|
| Honorary Fellows . . . . . | 10 |
| Fellows . . . . . | 92 |
| Fellows non-resident in British Isles . . . | 9 |
| Members . . . . . | 138 |
| Corporate Members . . . . . | 10 |
| Members non-resident in British Isles. . . | 22 |
| Corporate Members non-resident in British Isles . | 2 |
| Student Members . . . . . | 2 |
| TOTAL . . . | 285 |

## TREASURER'S REPORT FOR THE YEAR 1924

THE receipts for the year are £34 greater than for last year and, allowing for life compositions, the true income is £66 greater, due to increased receipts from subscriptions and from invested funds. The income from subscriptions will not be maintained unless the membership increases, as there is less money owing from members in arrear with their subscriptions than has been the case of late years.

The expenses are down by £82, due mainly to a reduction of £62 in the cost of printing the *Transactions*. The net result is a balance of income over expenditure of £325. 17*s*. 11*d*. The expenditure is likely to be greater in 1925 as the Council have decided to issue a special index of the *Transactions*, 25 volumes of which have now been published.

A further £500 has been invested in War Loan. The accumulated funds account stands at practically £800, in addition to the Life composition fund of £52. 10*s*. 0*d*.

The books and manuscripts of the Society are insured against loss by fire with the Guardian Assurance Co., Ltd., for the sum of £300, but they are not included in the assets of the Society shown in the balance sheet.

E. O. HENRICI,
Hon. Treasurer.

10*th January*, 1925.

## INCOME AND EXPENDITURE ACCOUNT FOR YEAR ENDING 31ST DECEMBER, 1924

| *Dr* *Expenditure* | £ | s. | d. | *Income* | £ | s. | d. | £ | s. | *Cr* d. |
|---|---|---|---|---|---|---|---|---|---|---|
| To Printing and Stationery | 463 | 0 | 0 | By Subscriptions: | | | | | | |
| Clerical Assistance | 98 | 17 | 3 | Fellows, arrears | 29 | 18 | 6 | | | |
| Postage and receipt stamps | 30 | 16 | 0½ | ,, for 1924 | 309 | 15 | 0 | | | |
| Sundries | 25 | 6 | 4½ | Members, arrears | 50 | 5 | 0 | | | |
| Advertising | 15 | 15 | 6 | ,, for 1924 | 323 | 5 | 0 | | | |
| Physical Society—share of Exhibition expenses | 8 | 6 | 8 | Corporate members for 1924 | 32 | 11 | 0 | | | |
| Life composition account | 10 | 10 | 0 | Students for 1924 | 2 | 2 | 0 | | | |
| Balance—income over expenditure | 325 | 17 | 11 | Entrance fees | 10 | 10 | 0 | | | |
| | | | | | 758 | 6 | 6 | | | |
| | | | | *Less* rebate to members of the Institute of Physics | 14 | 14 | 3 | | | |
| | | | | | | | | 743 | 12 | 3 |
| | | | | Life compositions | | | | 10 | 10 | 0 |
| | | | | Sales of publications | 156 | 15 | 9 | | | |
| | | | | *less* scarce *Transactions* purchased | 5 | 19 | 6 | | | |
| | | | | | | | | 150 | 16 | 3 |
| | | | | Advertisements in *Transactions*, less expenses | | | | 38 | 15 | 10 |
| | | | | Interest on investments and deposit | | | | 34 | 15 | 5 |
| | £978 | 9 | 9 | | | | | £978 | 9 | 9 |

## BALANCE SHEET AT 31ST DECEMBER, 1924

| *Liabilities* | £ | s. | d. | £ | s. | d. | *Assets* | £ | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|
| To Sundry Creditors | | | | 138 | 13 | 7 | By Sundry Debtors | 56 | 5 | 9 |
| Accumulated Funds: | | | | | | | Cash at Bank | 17 | 9 | 0 |
| Balance at 31st Dec. 1923 | 474 | 1 | 6½ | | | | Petty cash in hand | 9 | 18 | 0½ |
| *Plus* income over expenditure | 325 | 17 | 11 | | | | Investments at cost*: | | | |
| | | | | 799 | 19 | 5½ | £100 Grand Trunk Western Railway 4 % 1st Mortgage Gold Bonds | 98 | 10 | 0 |
| Life composition fund | | | | 52 | 10 | 0 | £800 War Stock 1929/47 | 809 | 0 | 8 |
| | | | | £991 | 3 | 0½ | | £991 | 3 | 0½ |

* *Mem.* Value of investments, market price, 31st Dec. 1924 is £893. 10*s.* 0*d.*

We, the undersigned Auditors elected at the General Meeting held on 11th December, 1924, do hereby certify that we have examined the above accounts and do find them correct.

T. F. CONNOLLY,
W. H. A. FINCHAM,
Hon. Auditors.

E. O. HENRICI,
Hon. Treasurer.

31*st January*, 1925.